A Tutorial on State of Charge Estimation Using the 3rd-Order R-RC Model

CASE STUDY
MARCH 2017

BY
HAMED AFSHARI
CENTRE FOR MECHATRONICS & HYBRID TECHNOLOGIES
224-200 LONGWOOD ROAD SOUTH
HAMILTON, ON
L8P 0A6, CANADA

A Tutorial on State of Charge Estimation Using the 3rd-Order R-RC Model

Abstract

This tutorial describes the process for the state of charge (SOC) estimation of Li-Ion cells using an equivalent circuit model. It helps students create and run a SOC estimation strategy based on the 3rd-order R-RC model in MATLAB-Simulink. The tutorial starts with a general overview of state estimation using the extended Kalman filter (EKF) [1], and the novel smooth variable structure filter (SVSF) [2] method. Thereafter, it presents applications of the EKF and the SVSF method for SOC estimation of a Li-Ion cell using the 3rd-order R-RC model and input-output (current-terminal voltage) data. Finally, it compares the performance of these estimators where the 3rd-order R-RC model is subjected to uncertainties on the initial SOC estimation, and parameters of the model.

1. Introduction

State estimation is the process of extracting the value of a hidden state from indirect, inaccurate and uncertain measurements of the system. Model-based state estimation methods are developed based on the system's mathematical model representing its dynamics. The main goal of the state estimation task is to minimize the state estimation error while being robust to parametric and modeling uncertainties, noise and perturbations. Noise and perturbations are inherently present in the measurement process, and are caused by instruments and environmental factors. System uncertainties are usually caused by inaccuracies in modeling, approximations, nonlinearities, discretization errors, and variations in physical parameters of the system.

In model-based state estimation methods, the filter recursively calculates the posterior probability density function (PDF) of the states based on the available information. Starting from a known prior PDF, the recursive algorithm includes two steps that are the prediction and the update stages. In the prediction stage, the system model is used to predict the state values. The predicted values of states are then refined and updated through the filter gain and based on new measurements from the system. The recursive equation of an estimated posteriori PDF may be calculated via an optimal form for an estimation problem with a linear state and measurement model subjected to Gaussian white additive noise. In such cases, the *a posteriori* PDF is expressed by simply using the mean and covariance [1].

The most popular method used to solve linear Gaussian state estimation problems is the Kalman filter. For general nonlinear and non-Gaussian systems, there are several techniques in the literature that are mainly based on linearization (e.g., the extended Kalman filter) or PDF approximation (e.g., the unscented Kalman filter, or the cubature Kalman filter). The main concerns with the Kalman filter is that it assumes that the system's model is known and that the noise is white. In real applications, however, there are considerable uncertainties that compromise these assumptions [1]. A new approach for robust state estimation, the smooth variable structure filter (SVSF) was introduced in 2007 [2]. The SVSF-type filtering is model-based but is more robust to uncertainties. This section summarizes the EKF and the SVSF method for state estimation.

1.1. The extended Kalman filter (EKF)

The EKF is used for estimating states of a nonlinear dynamic system. Local linearization is performed in this method in order to approximate the nonlinearity of the state or measurement model at the operating point and to calculate a corrective gain. Consider a dynamic system with a nonlinear state and measurement equation given by [1]:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}), \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k. \tag{2}$$

where $\mathbf{x}_k$, $\mathbf{u}_k$, and $\mathbf{z}_k$ are the state, control, and measurement vectors, respectively, and, $\mathbf{w}_k$ and $\mathbf{v}_k$ are the process uncertainty and measurement noise at time step *k*, respectively. It is assumed that **f**, **h** and **u** are known, where $\mathbf{w}_k$ and $\mathbf{v}_k$ are mutually independent white stochastic processes. The EKF derivation is based on the Taylor series expansion of the nonlinear state model (1) and measurement model (2) with linear terms. However, these nonlinear **f** and **h** functions cannot be applied to the covariance term directly, and their Jacobian's must be computed. The EKF has two main stages as follows:

The prediction step [1]**:**

- Calculation of the Jacobians for the state and measurement equations, **F** and **H**, respectively as follows:

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k}, \mathbf{u}_k}, \tag{3}$$

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k-1}}. \tag{4}$$

- Calculation of the predicted state and covariance estimates as follows:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}), \tag{5}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}. \tag{6}$$

The update step [1]:

- Determination of the innovation (or measurement error) and its covariance as:

$$\mathbf{e}_{\mathbf{z}_{k|k-1}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}), \tag{7}$$

$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k. \tag{8}$$

- Calculation of the sub-optimal EKF gain as follows:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1}. \tag{9}$$

- Updating the state and covariance estimates are given by:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{e}_{\mathbf{z}_{k|k-1}}, \tag{10}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T. \tag{11}$$

Fig. 1 shows a block-diagram scheme for a one cycle of the EKF method.

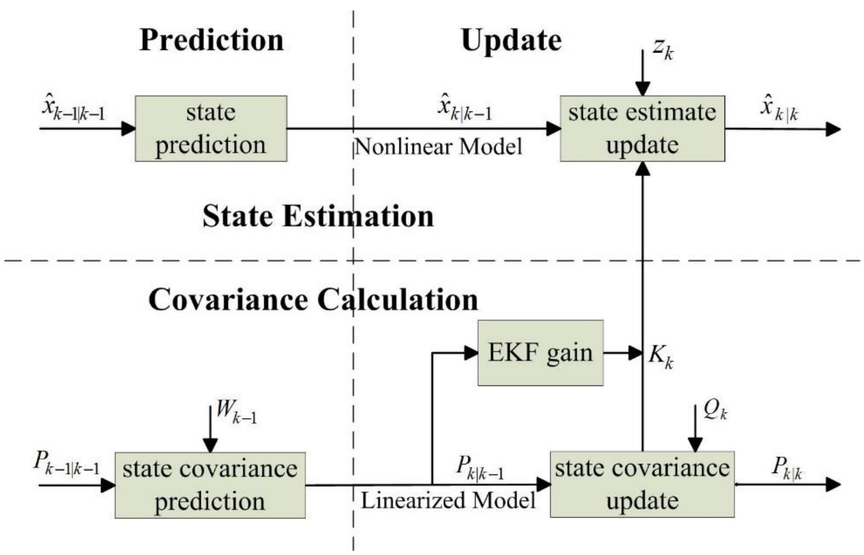


Fig. 1: Block-diagram scheme of a one cycle EKF for state estimation

Note that **Q** and **R** refer to the process and measurement noise covariance matrices, respectively. Numeric values of **Q** and **R** need to be tuned by trial and error in order to produce the best state estimation results. It is important to note that due to linearization, the EKF does not provide optimal state estimates in the mean square error sense. Additionally, the calculated error covariance matrix does not necessarily equal to the real error covariance matrix. The EKF's parameters need to be tuned such that the convergence improves. The convergence of the EKF is also dependent on the choice of the initial state estimates [1].

1.2. The smooth variable structure filter (SVSF)

The smooth variable structure filter (SVSF) is a new robust state estimation method that was introduced and implemented in 2007 [2]. This method is based on the variable structure system concept and produces robustness state estimates for systems with unknown but norm-bounded modeling and parametric uncertainties. The SVSF has a predictor-corrector structure and uses a discontinuous corrective gain to push state estimates towards their true values. Stability of the SVSF method with a discontinuous corrective gain is proven using a Lyapunov stability criterion. It was shown that if the stability criterion satisfied, then: $|e_{z_{k|k}}| < |e_{z_{k-1|k-1}}|$ [2].

The main concern with the SVSF method is eliminating the unwanted chattering effects from state estimates. Chattering is referred to as non-deterministic high-frequency oscillations resulting from the discontinuous corrective action of the SVSF gain. Habibi used a smoothing boundary layer to alleviate undesirable chattering effects. The implementation of the smoothing action is through a saturation function that interpolates the discontinuous corrective action with a smoothing boundary layer around the switching hyperplane. Outside the smoothing boundary layer the discontinuous correction is fully applied to maintain stability.

The SVSF method is recursive and may be applied to linear or nonlinear systems as [2]:

1. Prediction step [2]**:**

- Calculation of *a priori* state and measurement estimates:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{G}_{k-1}\mathbf{u}_{k-1}, \tag{12}$$

$$\hat{\mathbf{z}}_k = \mathbf{H}_k\mathbf{x}_k. \tag{13}$$

2. Update step [2]**:**

- Calculation of the SVSF's corrective gain that is stated as:

$$\mathbf{K}_k = \mathbf{H}^+\left(|\mathbf{e}_{\mathbf{z}_{k|k-1}}| + \boldsymbol{\gamma}|\mathbf{e}_{\mathbf{z}_{k-1|k-1}}|\right)\circ \operatorname{sgn}(\mathbf{e}_{\mathbf{z}_{k|k-1}}), \tag{14}$$

where *sgn* is the signum function, $\circ$ is the Schur product (element-by-element multiplication), and $\square^{+}$ is the pseudo-inverse transform. $\boldsymbol{\gamma}$ is also a diagonal matrix with positive entries such that $0<\gamma<1$. It denotes the convergence rate of the SVSF [2].

- Refine the *a priori* state estimate into the *a posteriori* state estimate:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k . \tag{15}$$

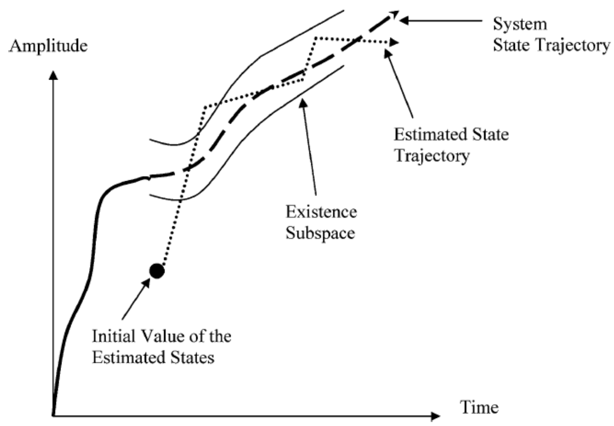


Fig. 2: Main concept of the SVSF method for estimation [2]

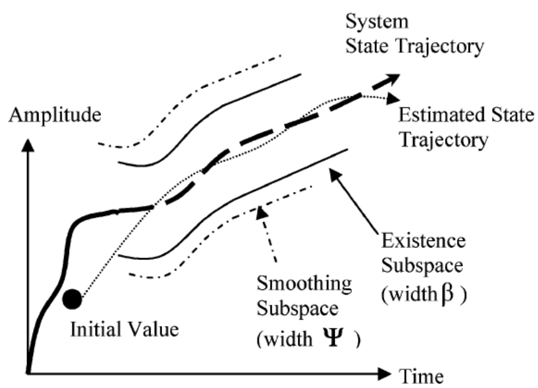
Fig. 2 presents the main concept of the SVSF method for state estimation. Note that in the SVSF derivation, it is assumed that the measurement noise and modeling uncertainties are norm-bounded such that $\|\mathbf{v}\|<V_{\max}$, and $\|\mathbf{w}\|<W_{\max}$. Furthermore, the discontinuous action of the filter corrective gain (14) generates high frequency chattering that degrades the estimation performance. In order to suppress the undesirable chattering effects from state estimates, a smoothing boundary layer is introduced into the filter formulation as follows [2]:

$$\mathbf{K}_k = \mathbf{H}^{+}\left(|\mathbf{e}_{\mathbf{z}_{k|k-1}}| + \boldsymbol{\gamma}|\mathbf{e}_{\mathbf{z}_{k-1|k-1}}|\right) \circ \mathrm{sat}(\boldsymbol{\psi}^{-1}\mathbf{e}_{\mathbf{z}_{k|k-1}}), \tag{16}$$

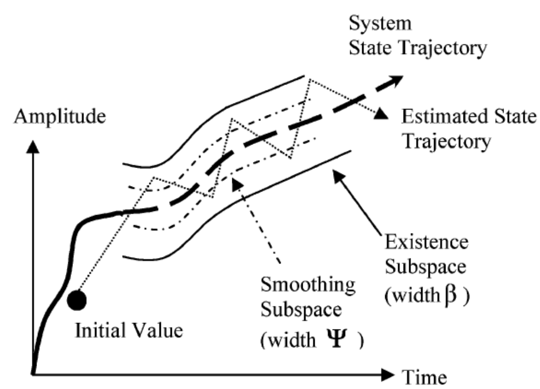
where $\boldsymbol{\psi}$ is a diagonal matrix with constant entries and denotes the smoothing boundary layer widths. In this context, the signum (i.e. *sgn*) function is replaced with a saturation (i.e. *sat*) function that causes the discontinuous action of the corrective gain being interpolated in the vicinity of the sliding hyperplane. Hence, outside the smoothing layer $\boldsymbol{\psi}$, the signum function applies to preserve stability. Otherwise, inside the smoothing layer $\boldsymbol{\psi}$, the saturation function applies to interpolate the signum function and supress chattering. The saturation function is defined by [2]:

$$\text{sat}(\psi_i^{-1} e_{z_{i,k|k-1}}) = \begin{cases} 1, & e_{z_{i,k|k-1}} / \psi_i > 1 \\ e_{z_{i,k|k-1}} / \psi_i & -1 \le e_{z_{i,k|k-1}} / \psi_i \le 1, \\ -1, & e_{z_{i,k|k-1}} / \psi_i \le -1 \end{cases} \tag{17}$$

Note that there are two different boundary layers in the SVSF concept including the existence layer, and the smoothing layer. The width of the existence layer is a function of the level of modeling uncertainties; the width of the existence layer varies in time and is unknown. The Smoothing boundary layer is design to encompass the existence subspace and by doing so, chattering is suppressed [2]. Fig. 3 presents the effect of the smoothing boundary layer width on the SVSF performance.



(a) SVSF for case with $\psi > \beta$



(b) SVSF for case with $\psi < \beta$

Fig. 3: Effect of the smoothing boundary layer width ψ on the SVSF performance [2]

2. The 3rd-order R-RC model for SOC Estimation

In order to implement a battery management system, the SOC needs to be estimated in real-time. State estimation methods are applied for extracting the SOC value based on terminal voltage measurements. In this case study, the 3rd-order R-RC model is used with the EKF and the SVSF method for SOC estimation. In this context, the applied current cycle is the same as what was used for modeling and parameter identification of the Li-Ion cell in the cases study 1. The terminal voltage is the only measurable variable. The performance of the EKF and the SVSF for SOC estimation is then compared under normal and uncertain conditions.

The 3rd-order R-RC model has four state variables including the voltage V_1 across the capacitor C_1, the voltage V_2 across the capacitor C_2, the voltage V_3 across the capacitor C_3, and the state of charge z. There is only one measurement variable that is the terminal voltage V_{Terminal}

across the two ends of the cell. The 3rd-order R-RC model may be represented in the state-space from as follows [3]:

$$\begin{bmatrix} V_{1,k} \\ V_{2,k} \\ V_{3,k} \\ z_k \end{bmatrix} = \begin{bmatrix} 1-\frac{\Delta t}{R_1C_1} & 0 & 0 & 0 \\ 0 & 1-\frac{\Delta t}{R_2C_2} & 0 & 0 \\ 0 & 0 & 1-\frac{\Delta t}{R_3C_3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k-1} \\ V_{2,k-1} \\ V_{3,k-1} \\ z_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{\Delta t}{C_1} \\ \frac{\Delta t}{C_2} \\ \frac{\Delta t}{C_3} \\ -\frac{\eta \Delta t}{C} \end{bmatrix} i_{k-1}, \tag{18}$$

$$V_{\text{Terminal},k} = OCV(z_k) - V_{1,k} - V_{2,k} - V_{3,k} - Ri_k, \tag{19}$$

where Δt is the sampling period, and η is the cell Columbic efficiency, and $OCV(z_k)$ is the open-circuit voltage relationship as a 10th-order polynomial function of the state of charge z_k. The input to the model is the current i_k and the output is the terminal voltage $V_{\text{Terminal},k}$. Moreover, *R* is the internal resistance of a cell. It has two values, R^+ for a positive input current, and R^- for a negative input current [3]. Fig. 4 shows a circuit diagram for the 3rd-order R-RC model.

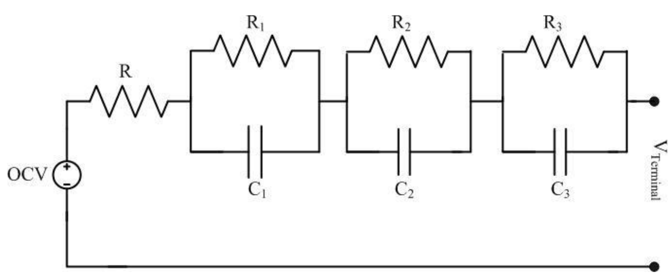


Fig. 4: The 3rd-order R-RC model of a battery cell

For simulating dynamics of the Li-Ion cell using the 3rd-order R-RC model, values of the eight unknown parameters $R_1, C_1, R_2, C_2, R_3, C_3, R^+$, and R^- are required. They can be obtained using an optimization technique and excitation cycles. Numeric values of the eight parameters are listed in Table 1. The measurement model for the 3rd-order R-RC model is nonlinear and needs to be linearized with respect to *z* using the Taylor's series expansion, whereas the high-order terms are neglected. The linearized form of the measurement equation is given by:

$$V_{t,k} = \left.\frac{\partial OCV(z_k)}{\partial z_k}\right|_{z_{k|k-1}} - V_{1,k} - V_{2,k} - V_{3,k} - R_0 i_k. \tag{20}$$

Table 1: Numeric values of parameters for the 3rd-order R-RC model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	1293.54 (Amp.s)
modeling resistance, R_1	0.00634 (Ohms)
modeling capacity, C_2	12384.35 (Amp.s)
modeling resistance, R_2	0.00624 (Ohms)
modeling capacity, C_3	4638.46 (Amp.s)
modeling resistance, R_3	0.00371 (Ohms)
internal resistance, R^+	0.03140 (Ohms)
internal resistance, R^-	0.02550 (Ohms)
sampling time, Δt	0.062 (s)

Note that in order to apply the EKF, the SVSF, or any other model-based estimator, the system's model should be observable. Observability is referred to as the ability to uniquely extracting states from measurements. Hence, if a system is not observable, this means that some of the state values cannot be uniquely obtained by using sensor measurements. There is a simple test for a linear systems to find out if it is observable or not. The observability test needs to be conducted as a preliminary step for designing a state estimator. To define the observability test, consider a dynamic system defined by a linear state and a linear measurement model, as follows:

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{G}\mathbf{u}_{k-1}, \tag{21}$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k, \tag{22}$$

where $\mathbf{F}$ and $\mathbf{H}_k$ denote the linearized state matrix, and the linearized measurement matrix, respectively. The observability matrix $\mathbf{O}_k$ is defined as:

$$\mathbf{O}_k = \begin{bmatrix} \mathbf{H}_k & \mathbf{H}_k\mathbf{F} & \cdots & \mathbf{H}_k\mathbf{F}^{n-1} \end{bmatrix}^T, \tag{23}$$

where n is the number of state variables. The systems is said to be completely observable, if the observability matrix is full-rank.

Note that the observability matrix for the 3rd-order R-RC model is time-varying. It is because the measurement matrix $\mathbf{H}$ calculated by linearization is time-varying. Hence, the rank of the observability matrix needs to be checked at each sample time. Fig. 5 presents the rank of the observability matrix (20) over time for the 3rd-order R-RC model. It is observed from Fig. 5 that the rank for the 3rd-order R-RC model is equal to four during the test. Since the model has four state variables, it is deduced that this model is completely observable.

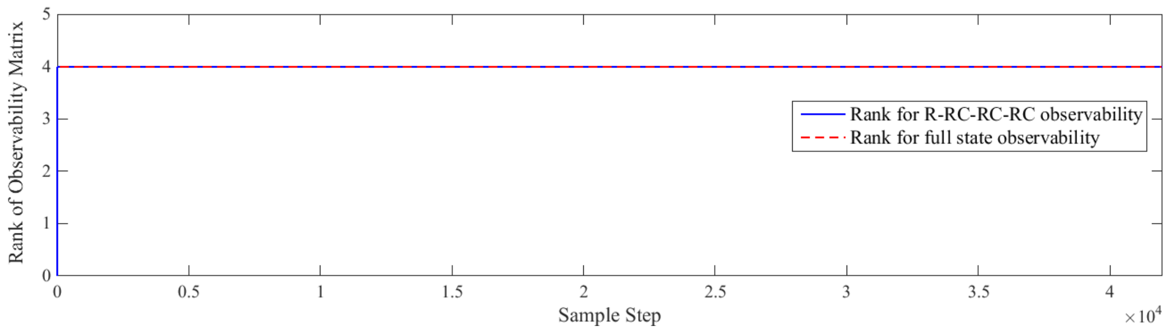


Fig. 5: Profile of the observability matrix's rank for the 3rd-order R-RC model

3. SOC estimation using EKF and SVSF methods

This section presents applications of the EKF and the SVSF method for SOC estimation of a Li-Ion cell using the 3rd-order R-RC model. It explains how the EKF and the SVSF blocks are created and run in Simulink.

3.1. SOC estimation using the EKF method

In order to estimate the SOC of a Li-Ion cell using the EKF method, a full-order state estimator is designed based on the 3rd-order R-RC model. For the EKF, the process noise covariance **Q** and the measurement noise covariance **R** are respectively set to:

$$\mathbf{Q} = \begin{bmatrix} 10^{-6} & 0 & 0 & 0 \\ 0 & 10^{-6} & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 10^{-6} \end{bmatrix}, \ \mathbf{R} = [5]. \tag{24}$$

Numerical values of **Q** and **P** have been calculated by trial and error in order to achieve the best performance for the EKF. The initial state error covariance matrix $\mathbf{P}_0$ and the initial state estimation error $\hat{\mathbf{x}}_0$ are assumed to be:

$$\mathbf{P}_0 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \ \hat{\mathbf{x}}_0(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 85 \end{bmatrix}. \tag{25}$$

Note that for the cell under the test, the actual initial SOC is equal to 90.7%.

The provided files include a Simulink model, named by SOCestimation_3rdRRCRCRC.slx. It contains the EKF and the SVSF block that are used to estimate the SOC based on the applied input current and the terminal voltage measurements. Fig. 6 presents a picture of the elements inside the Simulink model including the EKF and the SVSF block. Other supporting files include two MATLAB files named by Current.mat, and Voltage.mat. They contain the input current data and the measured terminal voltage data, respectively. To run Simulink models, all these files should be kept into a folder. Simulink files can only be run using MATLAB R2014b and probably newer versions. Parametric values for these models are fed into models using a mask interface built for Simulink models. The Simulink solver is set to "discrete" that is a fixed-step solver. The sample time for test and simulations is equal to 0.062 sec and the total running period is 2630.04 sec. To run the provided codes by MATLAB R2014b, the MATLAB's path directory needs to be set as the place where the folder contains Simulink models and other supporting files are located.

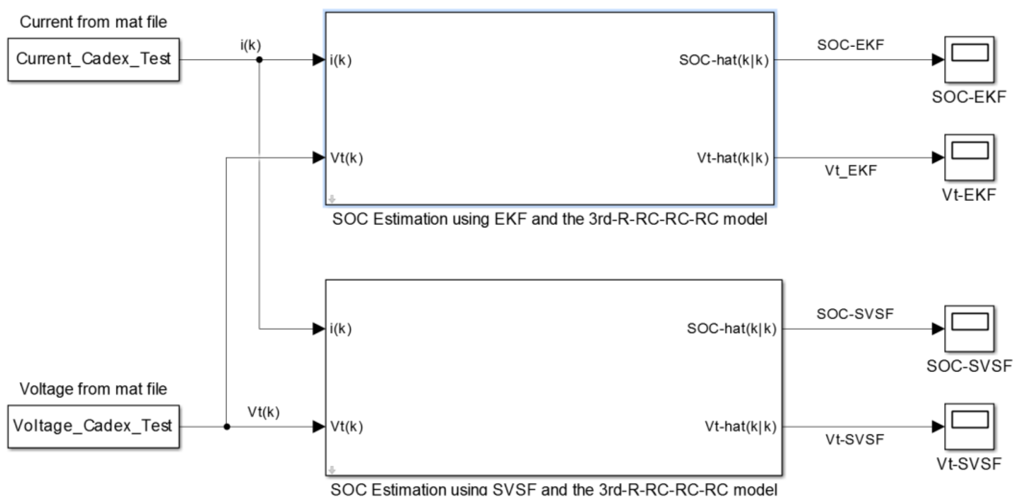


Fig. 6: The EKF and the SVSF blocks used for SOC estimation

The EKF block (the upper block in Fig. 6) has two inputs including the current file "Current.mat" and the terminal voltage measurement data "Voltage.mat" that are captured during the cycling test. These two mat files are fed into the Simulink model though "From Workspace" blocks that are located in "Sources". The "Current.mat" includes an array with two rows. The first row includes the time sequences and the second row includes the current sequences. Similarly, the "Voltage.mat" includes an array with two rows. The first row shows the time sequences and the second raw shows the voltage sequences. Moreover, there are two "Scopes" for the EKF blocks that are named by "SOC-EKF" and "Vt-EKF". The "Scope" block can be found in "Sinks". The two "Scope" blocks are used to plot the estimated SOC and the estimated terminal voltage generated by the EKF block.

Fig. 7 presents the user-interface input data, as required to run the EKF block for state of charge estimation. The first part of the input data includes physical parameters of the Li-Ion cell that are mainly obtained by parameter optimization, as listed in Table 1. The cell coulombic efficiency is assumed to be equal to one. The second part includes simulation parameters that are the simulation time, the initial state estimation vector, and the EKF design matrices including **Q**, **R**, and $\mathbf{P_0}$. The last part of the table asks for coefficients of the 10th-order polynomial function that approximates the open circuit voltage in terms of the state of charge variable. Numeric values of these coefficients are presented in Table 2 of the case study 1.

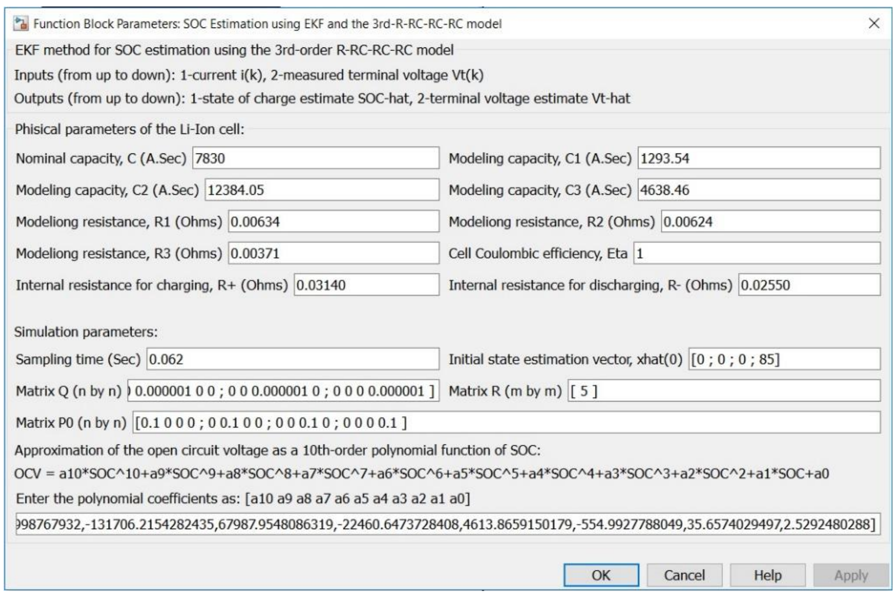


Fig. 7: Table of the user-interface inputs for the EKF block

Fig. 8 presents a picture of the main elements inside the EKF block. The EKF block is built using equations (3) through (11). The input current is multiplied by -1 in order to unify the current direction with the one used for obtaining state equations. Matrices **F** and **G** represent the state and the control matrix, respectively. A unit delay block is used at the top of Fig. 8 to integrate from $\hat{\mathbf{x}}_{k-1|k-1}$ and calculate $\hat{\mathbf{x}}_{k|k}$, where initial condition of the block is set to $\hat{\mathbf{x}}_0$. The OCV block uses the 10th-order polynomial to approximate the open circuit voltage in terms of the state of charge. A “Switch” function is used to select the value of the internal resistance between R^+ and R^- as a function of the current direction. In order to create the measurement error vector, $\mathbf{e}_{\mathbf{z}_{k|k-1}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})$, the predicted terminal voltage is subtracted from the measured one.

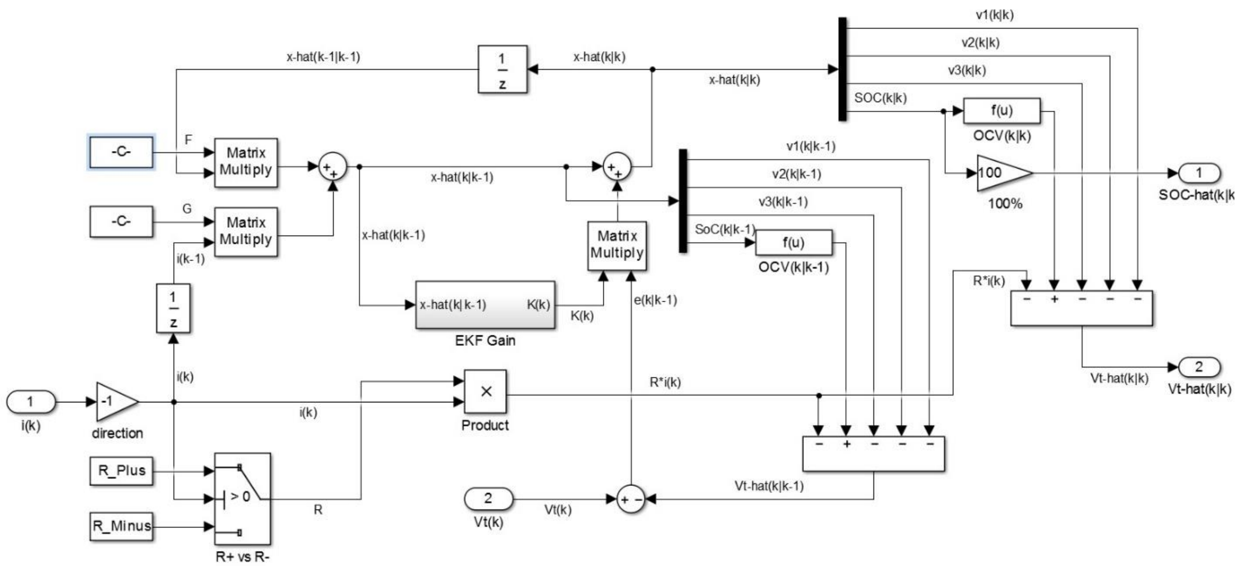


Fig. 8: A picture of the elements inside the EKF block

The EKF gain block is created based on equations (3) through (11). Fig. 9 presents a picture of the elements inside the EKF gain block. The input to this block is the predicted state vector that contains four entries. To separate the entries, a "Demux" block is used. According to equation (20), the first three entries of the measurement matrix **H** are equal to -1, and the fourth entry is calculated by $\partial OCV(z_k)/\partial z_k\big|_{z_{k|k-1}}$. The block d(OCV)/d(SOC) is designed to calculate the partial derivative of the OCV function with respect to the state of charge. This block applies a 9th-order polynomial function that is the partial derivative of the 10th-order OCV(SOC) function given by:

$$\partial OCV/\partial z_k = 10p_{10}z_k^{\,9} + 9p_9z_k^{\,8} + 8p_8z_k^{\,7} + 7p_7z_k^{\,6} + 6p_6z_k^{\,5} + 5p_5z_k^{\,4} + 4p_4z_k^{\,3} + 3p_3z_k^{\,2} + 2p_2z_k + p_1, \qquad (26)$$

where coefficients p_1 to p_{10} are listed in Table 3 of the case study 1. A "Matrix Concatenate" block and a "Reshape" block are used to build the measurement matrix $\mathbf{H} \in \mathbb{R}^{4\times 1}$.

Following Fig. 9, the state error covariance matrix is initially predicted using equation (6), and thereafter, it is updated using equation (11). The process noise matrix **Q** and the measurement noise matrix **R** are applied into the EKF block through the mask interface. The "Matrix Multiply" block is used to multiply the matrix or vector elements. It is important to check the matrix dimensions before multiplying the elements. Moreover, to calculate the transpose or the inverse of a matrix, there are two blocks in the Simulink library that are respectively named by "Transpose" and "Matrix Inverses". The output of the EKF gain is used to refine the *a priori* state estimate into the *a posteriori* state estimates.

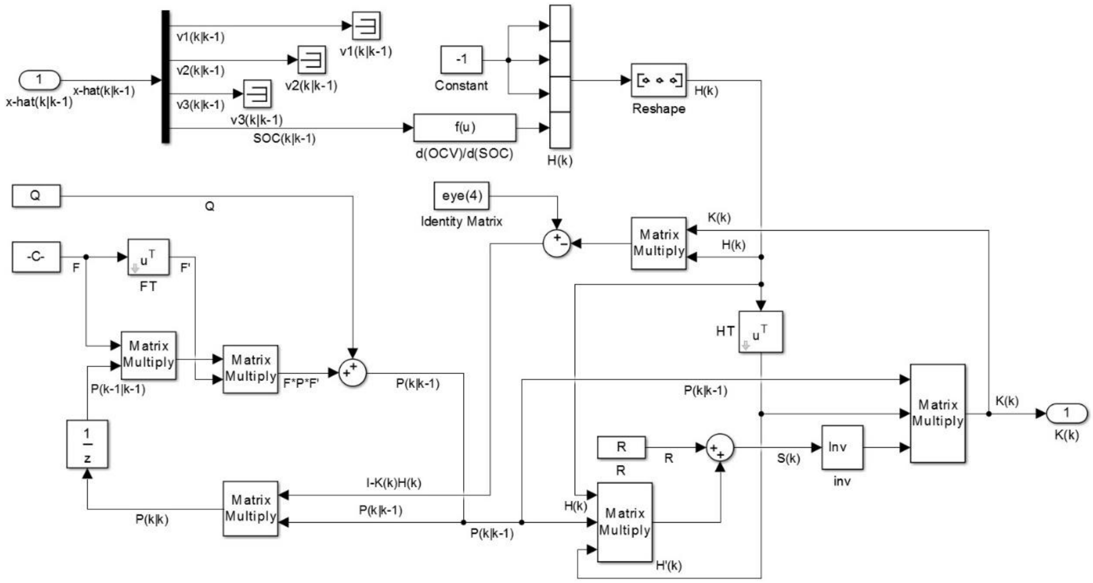


Fig. 9: A picture of the elements inside the EKF gain block

3.2. SOC estimation using the SVSF method

A full-order SVSF estimator is designed and implemented in order to estimate the SOC of the Li-Ion cell using the 3rd-order R-RC model. For the SVSF method, the convergence rate matrix $\boldsymbol{\gamma}$ and the smoothing boundary layer width $\boldsymbol{\psi}$ are respectively set to:

$$\boldsymbol{\gamma}=\begin{bmatrix}0.5 & 0 & 0 & 0\\ 0 & 0.5 & 0 & 0\\ 0 & 0 & 0.5 & 0\\ 0 & 0 & 0 & 0.5\end{bmatrix},\ \boldsymbol{\psi}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 5 & 0 & 0\\ 0 & 0 & 5 & 0\\ 0 & 0 & 0 & 5\end{bmatrix}. \qquad (27)$$

Numerical values of $\boldsymbol{\gamma}$ and $\boldsymbol{\psi}$ have been obtained by trial and error such that the SVSF presents its best performance for SOC estimation. Similar to the EKF method, the initial state estimation error vector $\hat{\mathbf{x}}_0$ are assumed to be:

$$\hat{\mathbf{x}}_0(0)=\begin{bmatrix}0 & 0 & 0 & 85\end{bmatrix}^T. \qquad (28)$$

Note that for the Li-Ion cell under the test, the initial SOC is equal to 90.7%.

The SVSF block is located at the lower side of SOCestimation_3rdRRCRCRC.slx. It is capable to estimate the SOC using the input-output (current-terminal voltage) data and the 3rd-order R-RC model. The SVSF block has two inputs that are the current data “Current.mat” and the output terminal voltage measurement data “Voltage.mat”. Moreover, it has two “Scopes” that are named by SOC-SVSF and Vt-SVSF. They are employed to plot the estimated SOC and the estimated terminal voltage profile. Fig. 10 shows the user-interface input data that are required to run the SVSF block. The first part of the input data includes physical parameters of the Li-Ion cell, as listed in Table 1. The cell coulombic efficiency is also assumed to be equal to one. The second part presents simulation parameters including the simulation time, the initial state estimation vector, and SVSF design matrices $\boldsymbol{\gamma}$ and $\boldsymbol{\psi}$. Similar to the EKF block, the last part of the table asks for coefficients of the 10th-order OCV-SOC polynomial function.

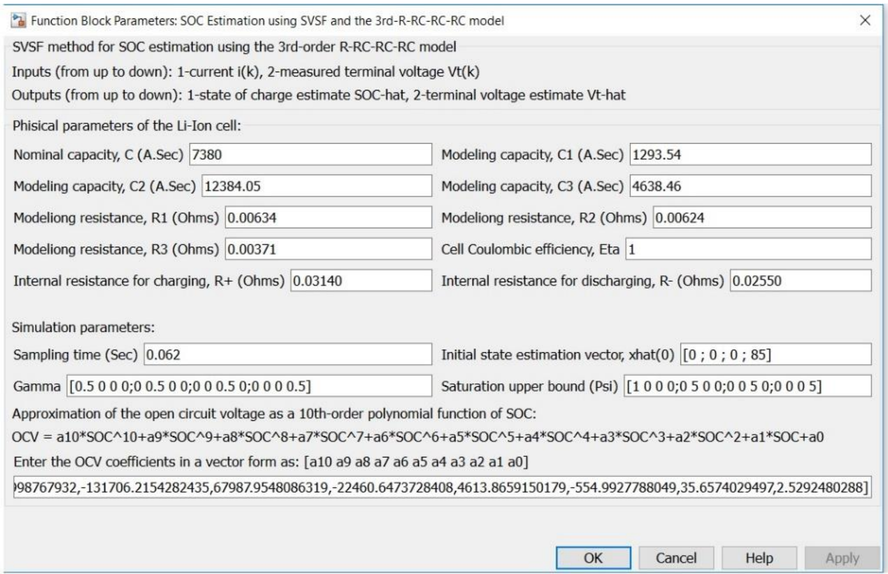


Fig. 10: Table of the user-interface inputs for the SVSF block

Fig. 11 shows a picture of the main elements inside the SVSF block, whereas the block is designed using equations (12) through (17). Similar to the EKF block, the input current is multiplied by -1 to unify the current direction with the one used by the state model. Matrix **F** and matrix **G** represent the state and the control matrix, respectively. A unit delay block is used at the top of Fig. 11 to integrate from $\hat{\mathbf{x}}_{k-1|k-1}$ and calculate $\hat{\mathbf{x}}_{k|k}$. Similar to the EKF, the initial condition for the unit delay block is set to $\hat{\mathbf{x}}_0$. The OCV block uses the 10th-order polynomial to calculate the open circuit voltage in terms of the state of charge. A "Switch" block is used to select the value of the internal resistance between R^+ and R^- based on the current direction. To calculate the *a priori* measurement error vector, $\mathbf{e}_{\mathbf{z}_{k|k-1}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})$, and the *a posteriori* measurement error vector, $\mathbf{e}_{\mathbf{z}_{k|k}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k})$, the *a priori* and the *a posteriori* terminal voltage estimates are subtracted from the measured terminal voltage $\mathbf{z}_k$, respectively.

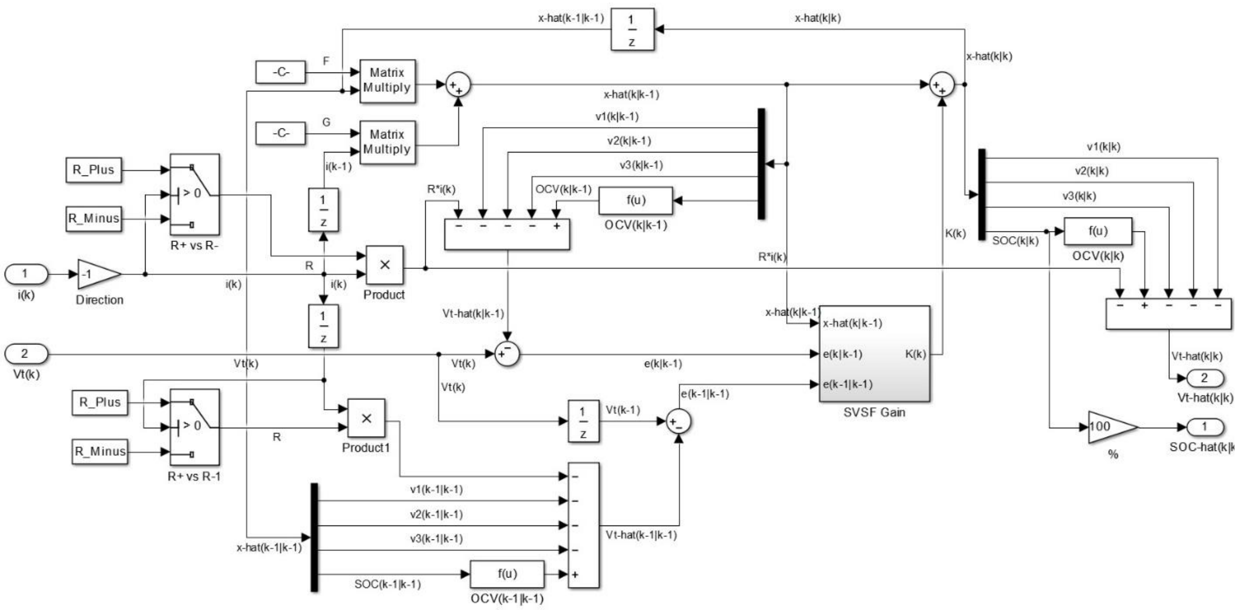


Fig. 11: A picture of the elements inside the SVSF block

The SVSF gain block is designed using equations (12) through (17). Fig. 12 shows a picture of the elements inside the SVSF gain block. The input to this block is the predicted state vector, and the *a priori* and the *a posteriori* measurement error. To separate entries of the predicted state vector, a "Demux" block is used. Similar to the EKF, the first three elements of **H** are equal to -1, and the fourth element is calculated by $\partial OCV(z_k)/\partial z_k\big|_{z_{k|k-1}}$. The block d(OCV)/d(SOC) uses a 9th-order polynomial function to calculate the partial derivative of the 10th-order OCV(SOC) with respect to the state of charge. A "Matrix Concatenate" block and a "Reshape" block are employed to build the measurement matrix $\mathbf{H} \in \mathbb{R}^{4\times 1}$. Thereafter, in order to build the SVSF gain formulation (16), the measurement matrix **H** is diagonized and its inverse is calculated. A "Saturation" block as well as two "Absolute" blocks are furthermore used to calculate the rest of the SVSF's gain formulation. The output of the block is the corrective gain of the SVSF that is used to update the predicted state estimation vector.

4. State estimation results

This section compares the SOC estimation results obtained for a Li-Ion cell under the normal and uncertain conditions. Under the normal condition, parameters of the equivalent circuit model are exactly known. Moreover, the cell's initial state of charge is approximately known. However, for the Li-Ion cell under uncertain conditions, there are uncertainties about parametric values of the model, as well as on the cell's initial SOC. The performance of the EKF and the SVSF for SOC estimation under the normal and uncertain conditions are also compared in terms of the accuracy and robustness.

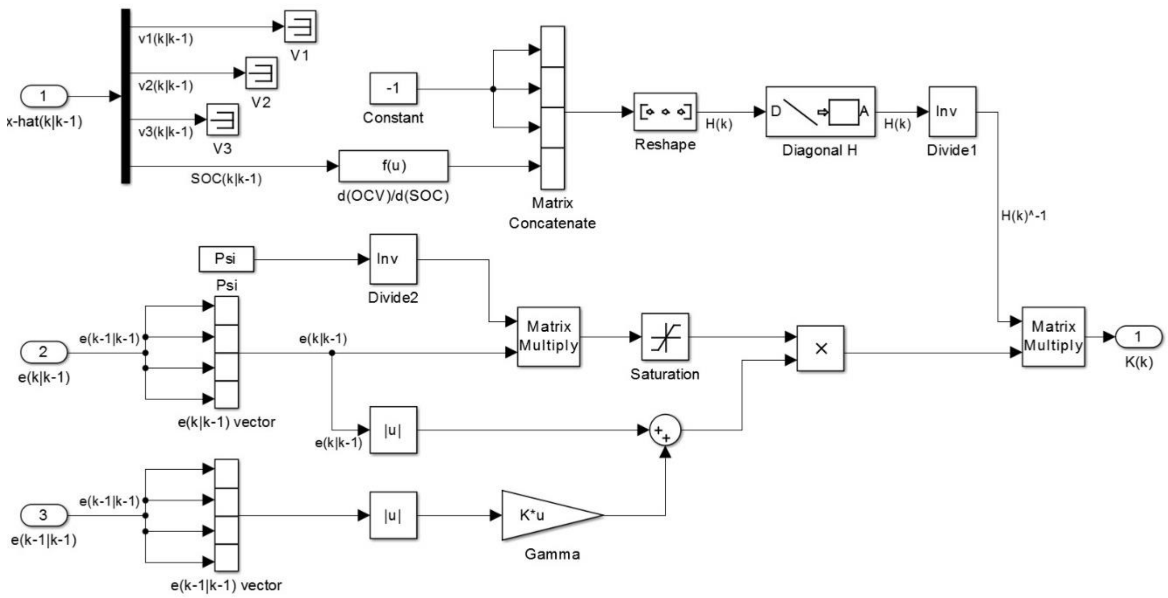


Fig. 12: A picture of the elements inside the SVSF gain block

4.1. SOC estimation under the normal condition

For a Li-Ion cell under the normal condition, parameters of the equivalent circuit model, and the initial SOC of the cell are known. Numeric values of parameters for the R-RC model are listed in Table 1. The initial SOC estimation is assumed to be equal to 85%, whereas the actual initial SOC is equal to 90.7% (obtained from the test data). The provided EKF and SVSF blocks are applied for SOC estimation under the normal condition. Fig. 13 compares the estimated SOC and the estimated terminal voltage with the actual ones. The actual SOC is directly obtained by coulomb counting (numerical integration), whereas the actual terminal voltage is obtained by voltmeter measurements. Furthermore, Table 2 shows RMS values of the error for the SOC estimation and the terminal voltage estimation using the 3^{rd}-order R-RC model.

Table 2: RMS values of the error for EKF and SVSF where initial SOC estimate = 85%

	SOC (%)	Terminal Voltage (v)
EKF	0.999	0.0238
SVSF	0.990	0.0225

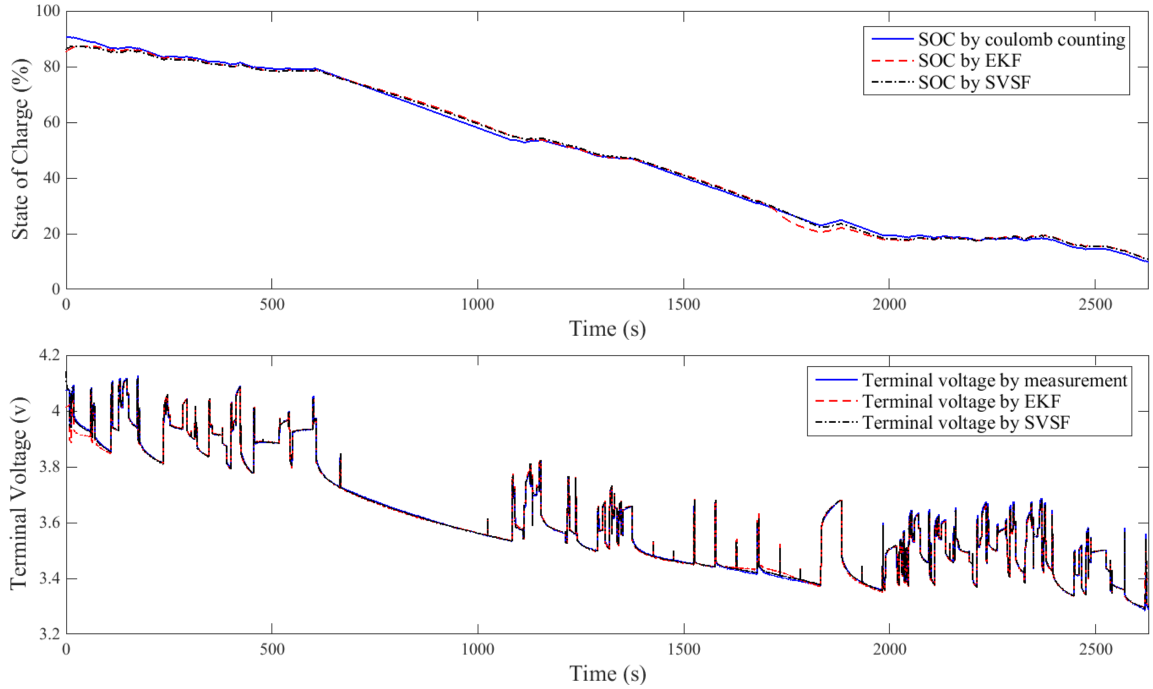


Fig. 13: Profiles of the actual and the estimated SOC and terminal voltage where initial SOC estimate = 85%

4.2. SOC estimation under uncertain conditions

There are several sources of noise, modeling and parametric uncertainties in the SOC estimation process. The main sources of modeling uncertainties include:

- Inaccuracies in the modeling due to approximation as an equivalent circuit model;
- Averaging of the OCV-SOC curve for charging and discharging with a single curve;
- Approximating the hysteresis curve;
- Uncertainties in the initial SOC; and
- Parametrization error; the main sources of parametric uncertainties include the error in the parameter identification of the cell, and deviations of battery's parameters from their nominal values due to aging.

There are also several sources for noise that include the instrumentation noise, the voltmeter measurement noise, and the unpredictable variations of the cell temperature.

In order to examine the robustness of EKF and the SVSF versus uncertainties, two uncertain scenarios are applied. In the first scenario, it is assumed that the initial SOC estimate is equal to 50%, whereas the actual initial SOC is 90.7%. In the second scenario, it is assumed that the nominal capacitance of the Li-Ion cell for estimators is equal to 5500 *Amp.s*, whereas the actual nominal capacitance is equal to 7830 *Amp.s*. Fig. 14 compares profiles of the estimated SOC and the estimated terminal voltage with the actual ones for the scenario with an uncertain initial SOC. Table 3 moreover compares the accuracy of the estimation methods in terms of RMS values of the errors for this scenario. Fig. 15 furthermore compares these profiles for the scenario with an uncertain nominal capacitance. Table 4 moreover compares the accuracy of the estimation methods in terms of RMS values of the errors for the second scenario. Note that values of the tuned parameters for each estimators remain as same as what were obtained for the normal scenario.

Table 3: RMS values of the error for EKF and SVSF where initial SOC estimate = 50%

	SOC (%)	Terminal Voltage (v)
EKF	3.684	0.0278
SVSF	5.458	0.0229

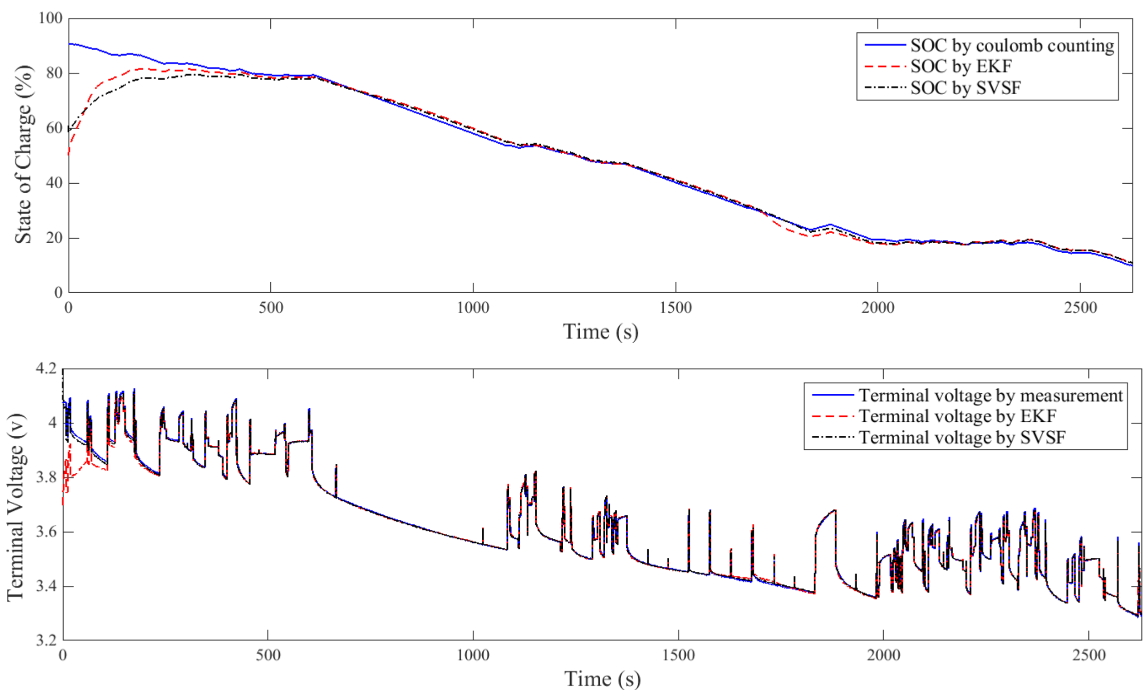


Fig. 14: Profiles of the actual and the estimated SOC and terminal voltage where initial SOC estimate = 50%

Table 4: RMS values of the error for EKF and SVSF where C_n = 5500 *Amp.s* for estimators

	SOC (%)	Terminal Voltage (v)
EKF	2.935	0.0281
SVSF	2.842	0.0225

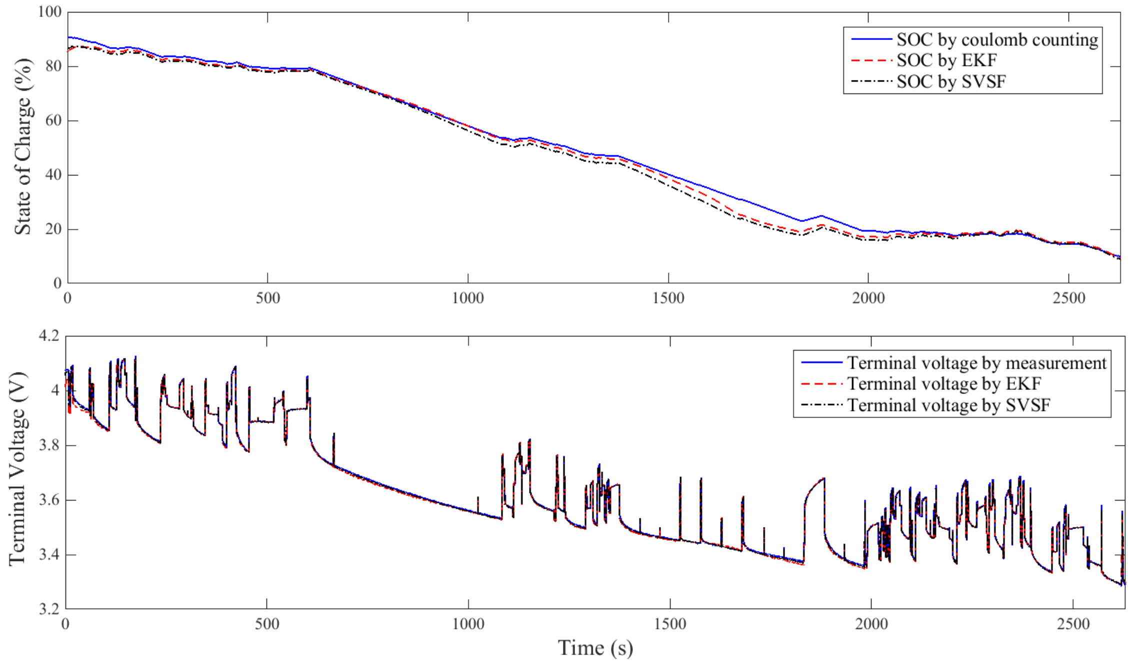


Fig. 15: Profiles of the actual and estimated SOC and terminal voltage where C_n = 5500 *Amp.s* for estimators

5. Conclusion

This case study presents applications of the extended Kalman filter (EKF) and the smooth variable structure filter (SVSF) for SOC estimation of a Li-Ion cell under the normal and uncertain conditions. Their accuracy and robustness for normal and uncertain scenarios were compared in term of the Root-Mean-Square (RMS) of the measurement error. In order to represent dynamics of the Li-Ion cell, a 3rd-order R-RC model (presented in the case study 1) is used. According to Figs. 13, 14 and 15, it is deduced that where there exist uncertainties in the initial condition, the EKF has a better performance over the SVSF for SOC estimation. But, where there exist uncertainties in modeling or parametric values, the SVSF has a better performance for. Moreover, for these scenarios the SVSF is more accurate for terminal voltage estimation.

References

[1] H. H. Afshari, S. A. Gadsden and S. H. Habibi, "Gaussian filters for parameter and state estimation: A general review of theory and recent trends," *Signal Processing*, vol. 135, p. 218–238, 2017.

[2] S. Habibi, "Smooth variable structure filter," *Proceedings of the IEEE*, vol. 95, no. 5, pp. 1026-1059, 2007.

[3] M. S. Farag, R. Ahmed, S. A. Gadsden, S. R. Habibi and J. Tjong, "A comparative study of Li-ion battery models and nonlinear dual estimation strategies," in *IEEE Transportation Electrification Conference and Expo (ITEC)*, Dearborn, MI, USA, 2012.