CASE STUDY #2: STATE OF CHARGE ESTIMATION USING THE EKF AND SVSF METHODS IN SIMULINK

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Contents

State Estimation

- **State estimation** is the process of extracting values of states from indirect, inaccurate and uncertain partial measurements of a system.
- The main objective is to minimize the **estimation error** that is the difference between the actual state values and the measured ones.
- **Modeling uncertainties** are caused by modeling inaccuracies, simplification assumptions, parametric variations, etc. **Measurement noise** are due to instrumental and environmental factors.
- Due to presence of measurement noise and modeling uncertainties, measurements do not reflect the actual state values.

State Space Models in Discrete-Time

- A dynamic system may be represented in state space by the process and the measurement model. The Process model is given by:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k),$$

where **x** is the state vector, **u** is the control vector, **w** is the vector of process uncertainties and **f** is the nonlinear state model.

- The measurement model is given by:

$$\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k),$$

where **z** is the measurement vector, **v** is the measurement noise, and **h** is the nonlinear measurement model.

- It is assumed that **w** and **v** are zero-mean white stochastic process and they are independent with respect to each other and the state vector **x.**

State Estimation Requirements

- **Requirements for model-based state estimation:**

1) System's state model

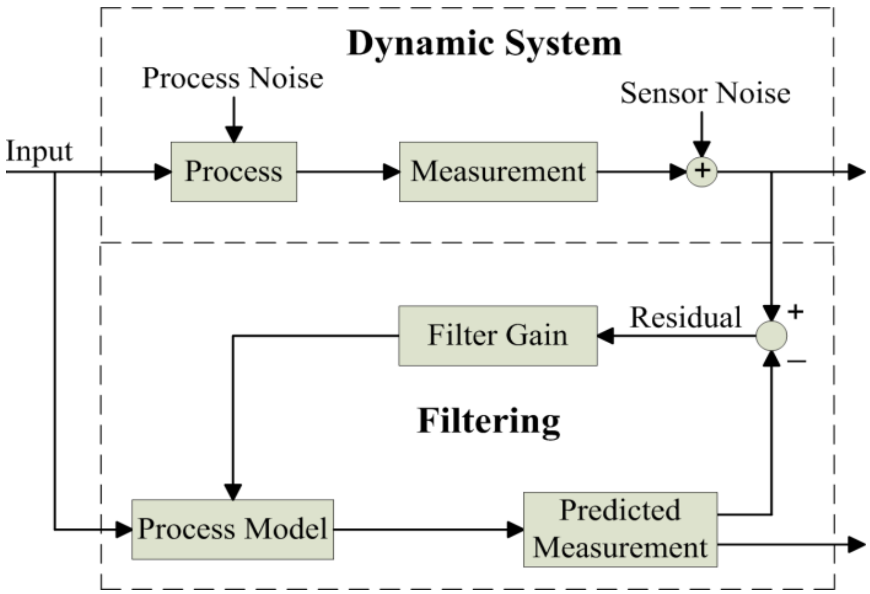
$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1})$$

2) System's measurement model

$$\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k)$$

3) The prior knowledge of the system

4) Input probabilistic characterization



Optimal Filtering vs. Robust Filtering

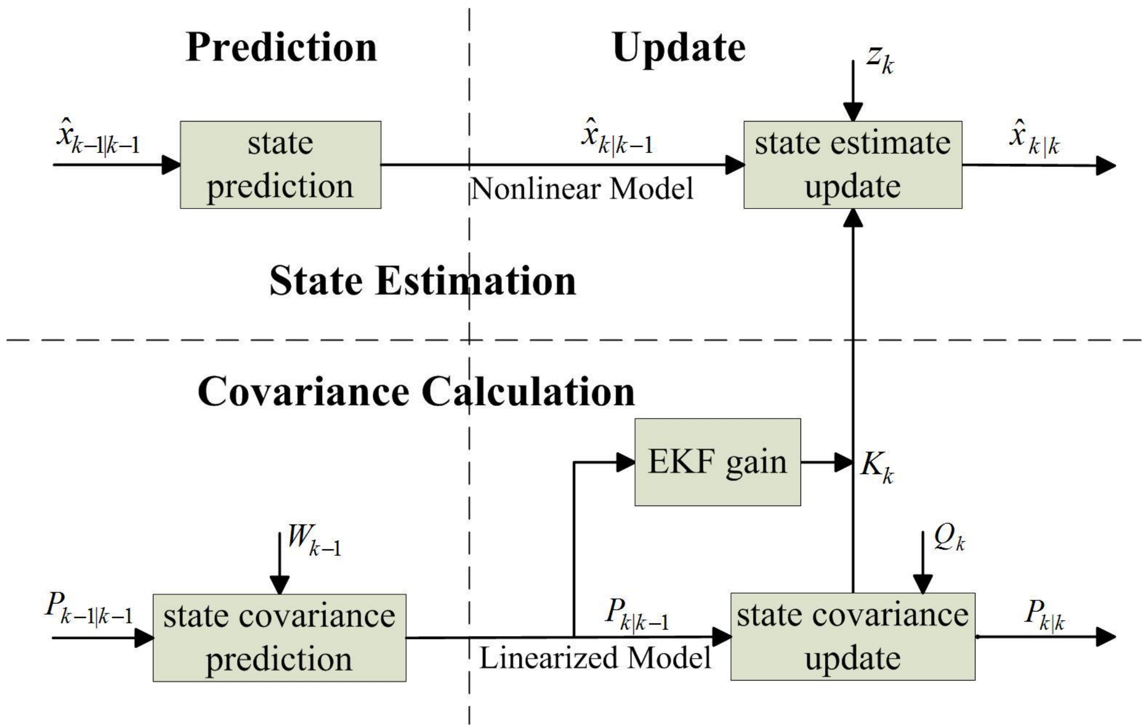
- **Main concerns with state estimation:**
  Presence of modeling inaccuracies, parametric uncertainties, small variations of parameters due to aging, discretization error, further to environmental and instrumental noise. There are two approaches:

1) **Optimal Filtering:**
   Provide estimates by minimizing the state estimation error with an exact knowledge of the process. The main method is the Kalman filter.

2) **Robust Filtering:**
   Design a filter that is insensitive to a wider range of noise and uncertainties. The main methods are the robust Kalman filter, H_∞ filter, and the novel smooth variable structure filter (SVSF).

Extended Kalman Filter (EKF)

- The extended Kalman filter uses the local linearization to estimate states of systems with nonlinear state and/or measurement models. The rest of the process is similar to the Kalman filter.



The Extended Kalman Filter (EKF)

- The EKF calculates Jacobians of the nonlinear state or measurement model based on Taylor series approximation and neglecting higher-order terms. The Jacobians are given by:

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k},\mathbf{u}_k}, \quad \mathbf{H}_k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k-1}}.$$

- Main steps of EKF:

1) Prediction: $$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}), \quad \mathbf{P}_{k|k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}.$$

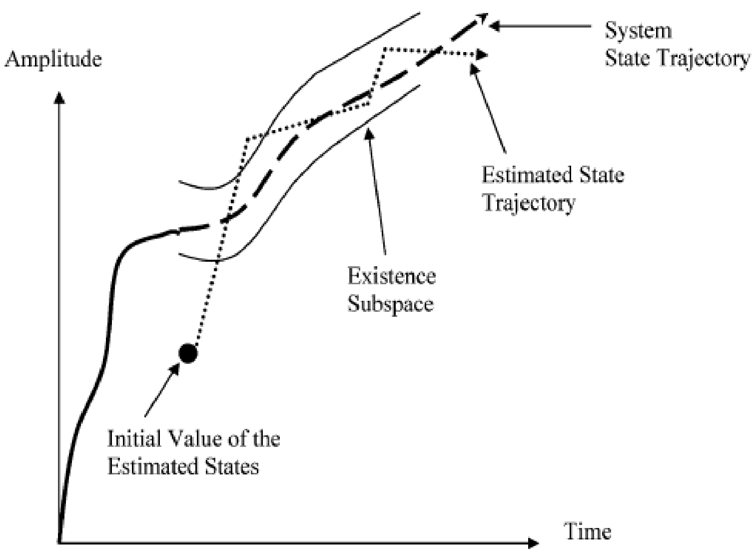
2) Innovation calculation: $$\mathbf{e}_{\mathbf{z}_{k|k-1}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}), \quad \mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k.$$

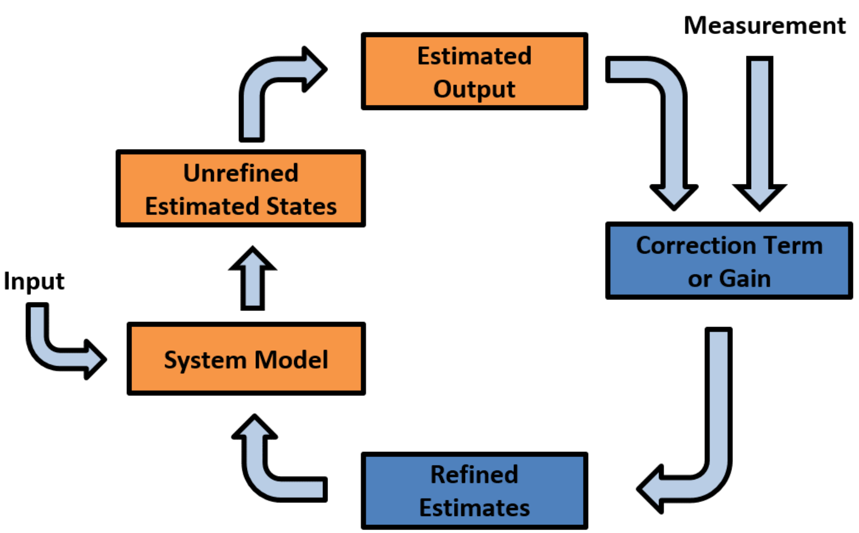
3) Gain calculation: $$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1}.$$

4) Update: $$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{e}_{\mathbf{z}_{k|k-1}}, \quad \mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T.$$

Smooth Variable Structure Filter (SVSF)

- The SVSF shows robustness against modeling uncertainties with a comparable accuracy compares to the Kalman-type filtering.
- The SVSF is designed based on the variable structure system concept and is formulated in a **predictor-corrector** from as follows:





Main Steps of the SVSF

1) Prediction of the states and measurements:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}), \quad \hat{\mathbf{z}}_{k|k-1} = \mathbf{H}\hat{\mathbf{x}}_{k|k-1}.$$

2) Calculation of the measurement error:

$$\mathbf{e}_{\mathbf{z}_{k|k-1}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}), \quad \mathbf{e}_{\mathbf{z}_{k|k}} = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k}).$$

3) Calculation of the corrective gain:

$$\mathbf{K}_k = \mathbf{H}^{+}(|\mathbf{e}_{\mathbf{z}_{k|k-1}}| + \boldsymbol{\gamma}|\mathbf{e}_{\mathbf{z}_{k-1|k-1}}|) \circ \mathrm{sgn}(\mathbf{e}_{\mathbf{z}_{k|k-1}}),$$

4) Update the state estimate as:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k.$$

- To suppress chattering, a **smoothing boundary layer** is applied:

$$\mathbf{K}_k = \mathbf{H}^{+}(|\mathbf{e}_{\mathbf{z}_{k|k-1}}| + \boldsymbol{\gamma}|\mathbf{e}_{\mathbf{z}_{k-1|k-1}}|) \circ \mathrm{sat}(\boldsymbol{\psi}^{-1}\mathbf{e}_{\mathbf{z}_{k|k-1}}),$$

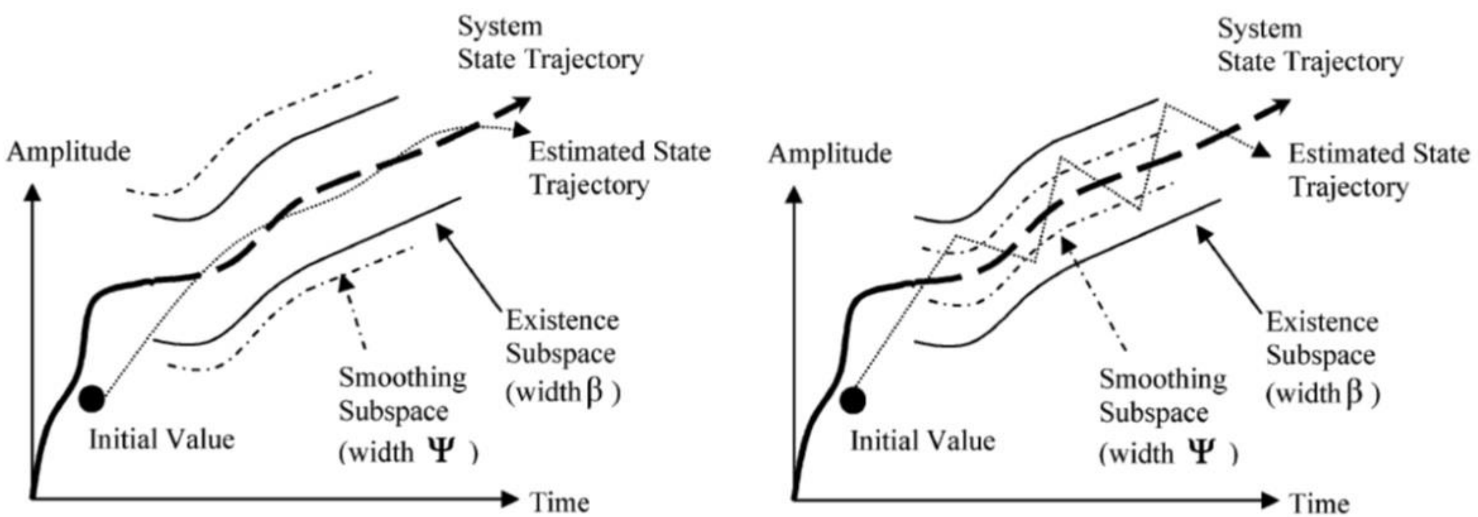
SVSF Design Notes

- Regarding the SVSF gain formulation, sgn is the signum function, $\circ$ is the Schur product (element-by-element multiplication), and $\square^{+}$ is the pseudo-inverse transform. $\boldsymbol{\gamma}$ is a diagonal matrix with positive entries: $0 < \gamma < 1$.
- $\boldsymbol{\psi}$ is a diagonal matrix with constant entries and denotes the smoothing boundary layer widths.
- To suppress chattering from state estimates, a **smoothing boundary layer** is applied. In this context, the signum (sgn) function is replaced with a saturation (sat) function and this interpolates the discontinuous corrective action of the SVSF gain.
- The sat function is defined by:

$$\mathrm{sat}(\psi_i^{-1} e_{z_{i,k|k-1}}) = \begin{cases} 1, & e_{z_{i,k|k-1}} / \psi_i > 1 \\ e_{z_{i,k|k-1}} / \psi_i & -1 \le e_{z_{i,k|k-1}} / \psi_i \le 1, \\ -1, & e_{z_{i,k|k-1}} / \psi_i \le -1 \end{cases}$$

Smoothing Boundary Layer Concept

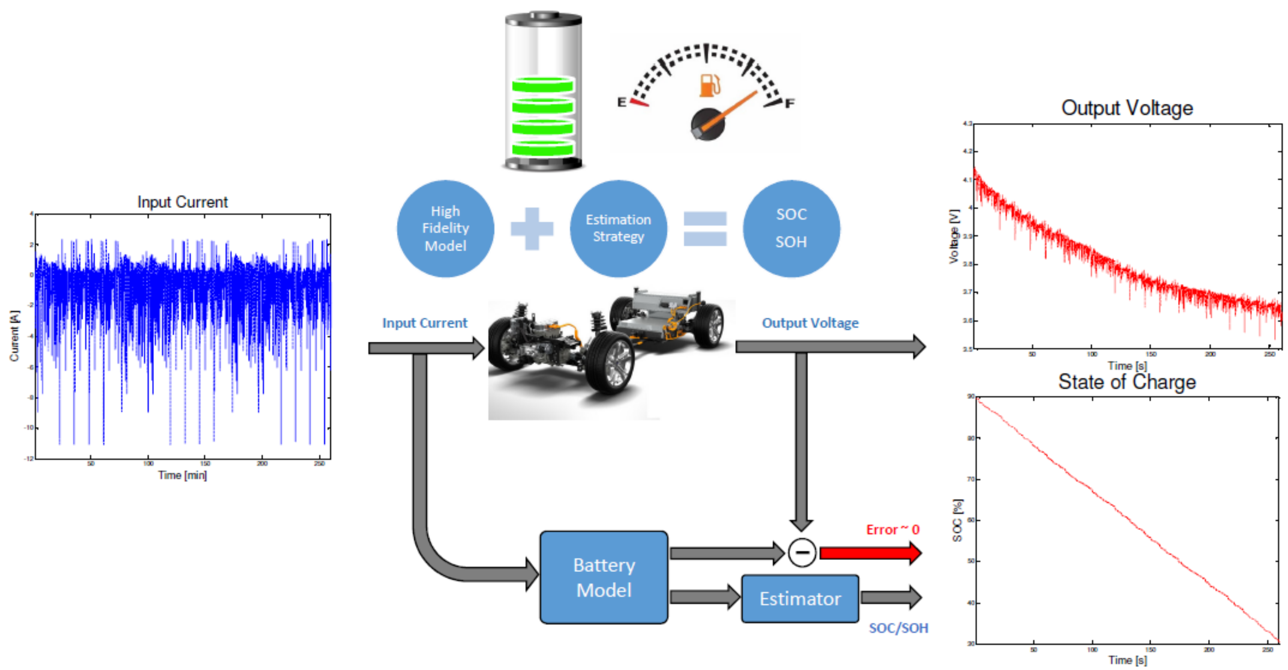
- SVSF has two boundary layers: 1-the existence layer, 2-the smoothing layer.
- The existence layer’s width is unknown and time-varying. It is a function of modeling uncertainties. The smoothing layer is design to encompass the existence subspace and by doing so, chattering is suppressed.



(a) SVSF for case with $\psi > \beta$ (b) SVSF for case with $\psi < \beta$

Importance of SOC estimation

- In spite the amount of fuel in tank that is measured using a fuel gage, there is not any direct way to measure the state of charge of a cell.
- The SOC needs to be estimated using a state estimator and input-output data:



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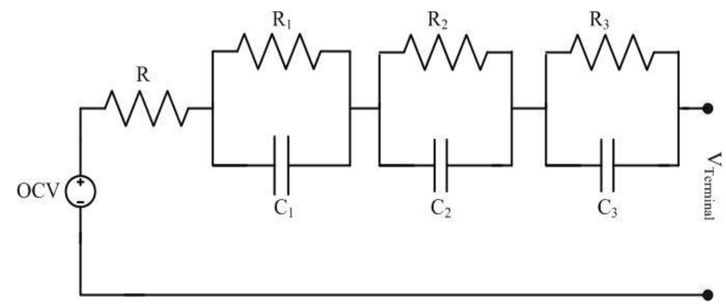
The Third-Order R-RC Model

- In order to design a estimator, the process needs to accurately be modeled.
- The R-RC-RC-RC model uses three R-C elements for representing cell dynamics:



$$\begin{bmatrix} V_{1,k+1} \\ V_{2,k+1} \\ V_{3,k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\frac{\Delta t}{R_1 C_1} & 0 & 0 & 0 \\ 0 & 1-\frac{\Delta t}{R_2 C_2} & 0 & 0 \\ 0 & 0 & 1-\frac{\Delta t}{R_3 C_3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k+1} \\ V_{2,k+1} \\ V_{3,k+1} \\ z_{k+1} \end{bmatrix} + \begin{bmatrix} \frac{\Delta t}{C_1} \\ \frac{\Delta t}{C_2} \\ \frac{\Delta t}{C_3} \\ -\frac{\eta \Delta t}{C} \end{bmatrix} i_k, \quad V_{t,k} = OCV(z_k) - V_{1,k} - V_{2,k} - R_0 i_k,$$

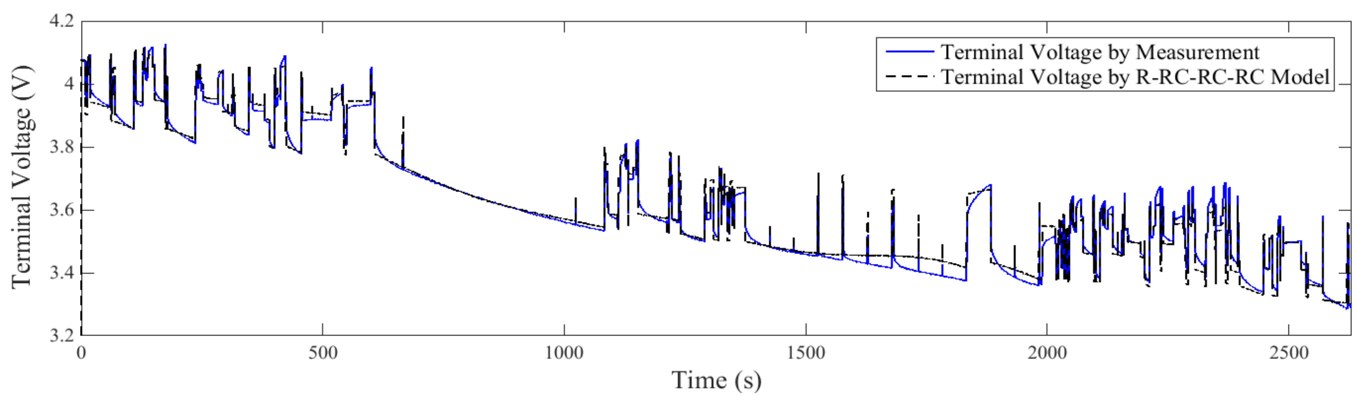
- Values of C_1, R_1, C_2, R_2, C_3, R_3, $R+$, and $R-$ were obtained by capturing input-output data and applying the genetic algorithm.

Parameters of 3rd-Order R-RC Model

- Numeric values of parameters obtained for 3rd-order R-RC model using optimization:

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	1293.54 (Amp.s)
modeling resistance, R_1	0.00634 (Ohms)
modeling capacity, C_2	12384.35 (Amp.s)
modeling resistance, R_2	0.00624 (Ohms)
modeling capacity, C_3	4638.46 (Amp.s)
modeling resistance, R_3	0.00371 (Ohms)
internal resistance, R_0^+	0.03140 (Ohms)
internal resistance, R_0^-	0.02550 (Ohms)
sampling time, Δt	0.062 (s)

- Comparison between the optimized and the actual terminal voltage:



State of Charge (SOC) Estimation

- Implementing the EKF and the SVSF for SOC estimation
- Observability of a model can be checked via the observability matrix:

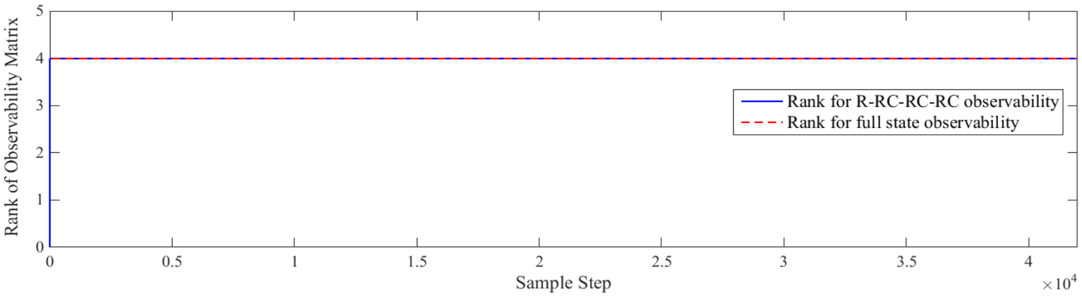
$$\mathbf{O} = \begin{bmatrix} \mathbf{H} & \mathbf{HF} & \cdots & \mathbf{HF}^{n-1} \end{bmatrix}^T,$$

  The systems is said to be completely observable, if the observability matrix **O** is full-rank.

- The 3rd-order R-RC model is completely observable, since the rank of the observability matrix O is equal to 4 (the number of states).

- Rank of the 3rd-order R-RC:

  (Proper for SOC estimation)



Linearization of Measurement Model

- To use the EKF and the SVSF method for SOC estimation of the Li-Ion cell, the measurement model needs to be linearized with respect to the state of charge variable z_k.
- The linearization is performed using the Taylor's series expansion, where high-order terms are neglected. The Linearized model is:

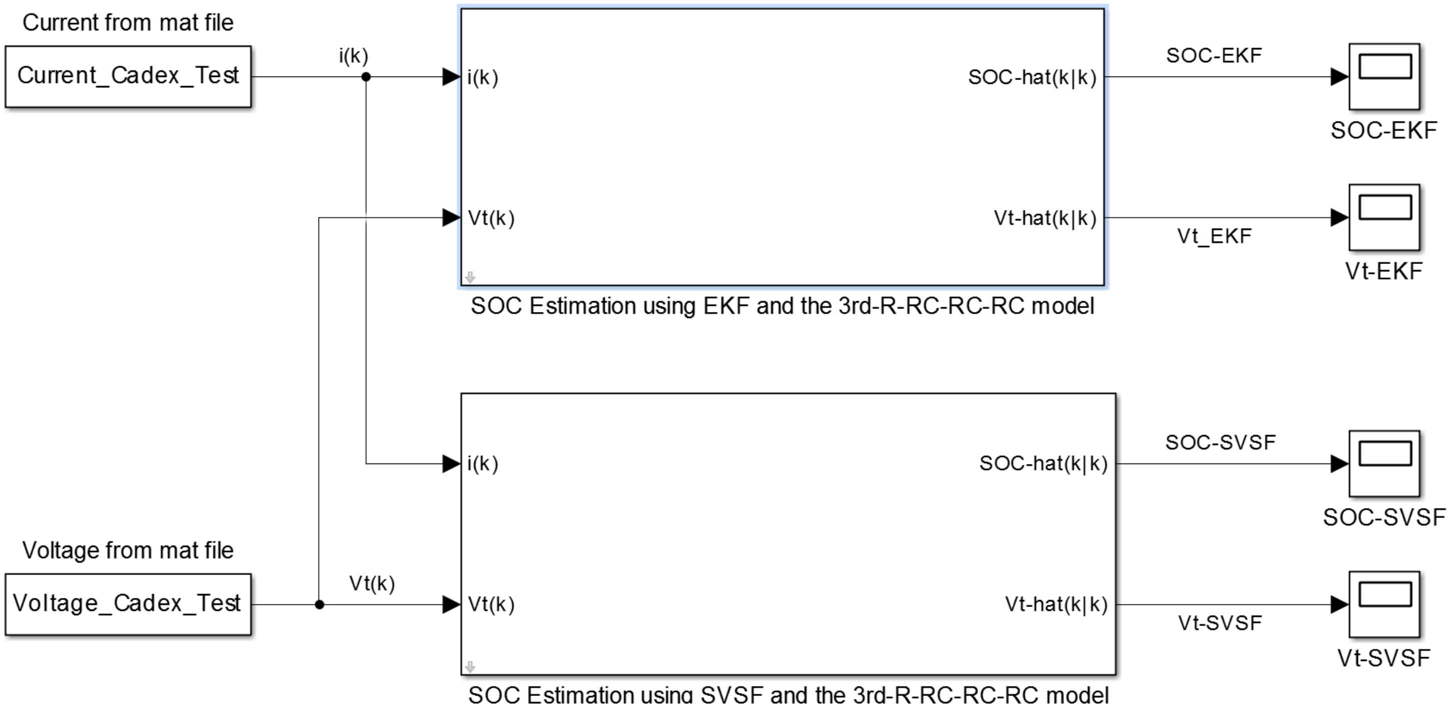
$$V_{t,k} = \left.\frac{\partial OCV(z_k)}{\partial z_k}\right|_{z_{k|k-1}} - V_{1,k} - V_{2,k} - V_{3,k} - R_0 i_k .$$

- The main nonlinear term is the first (the partial derivative) term:

$$\left.\frac{\partial OCV}{\partial z_k}\right|_{z_{k|k-1}} = 10p_{10}{z_k}^9 + 9p_9{z_k}^8 + 8p_8{z_k}^7 + 7p_7{z_k}^6 + 6p_6{z_k}^5 + 5p_5{z_k}^4 + 4p_4{z_k}^3 + 3p_3{z_k}^2 + 2p_2 z_k + p_1\Big|_{z_{k|k-1}}$$

Review of the Simulink Model

- The EKF and the SVSF blocks for SOC estimation using Simulink:



SOC Estimation Using EKF

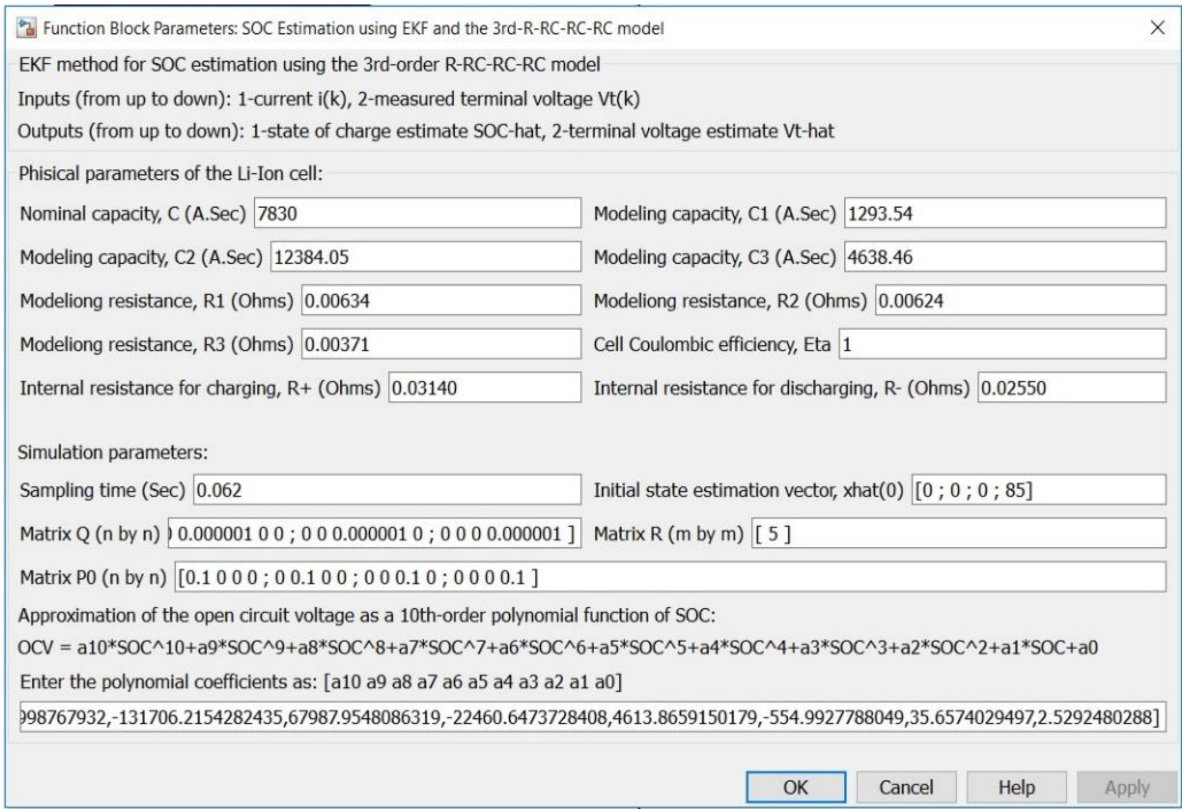
- For the EKF, the process noise covariance **Q** and the measurement noise covariance **R** are respectively set to:

$$\mathbf{Q}=\begin{bmatrix}10^{-6}&0&0&0\\0&10^{-6}&0&0\\0&0&10^{-6}&0\\0&0&0&10^{-6}\end{bmatrix},\ \mathbf{P}_0=\begin{bmatrix}0.1&0&0&0\\0&0.1&0&0\\0&0&0.1&0\\0&0&0&0.1\end{bmatrix},\ \hat{\mathbf{x}}_0(0)=\begin{bmatrix}0\\0\\0\\85\end{bmatrix},\ \mathbf{R}=[5].$$

- Numeric values of **Q, R**, and $\mathbf{P_0}$ have been calculated by trial and error in order to achieve the best performance for the EKF.
- The actual initial SOC is about 90.7%. The actual values of $V_1(0)$, $V_2(0)$, $V_3(0)$ are unknown. They are assumed to be zero.
- The EKF block has two inputs that are the current data “Current.mat” and the measured terminal voltage data “Voltage.mat”.

The EKF User-Interface Window

- The EKF user-interface window for input data:


Function Block Parameters: SOC Estimation using EKF and the 3rd-R-RC-RC-RC model

EKF method for SOC estimation using the 3rd-order R-RC-RC-RC model

Inputs (from up to down): 1-current i(k), 2-measured terminal voltage Vt(k)

Outputs (from up to down): 1-state of charge estimate SOC-hat, 2-terminal voltage estimate Vt-hat

Phisical parameters of the Li-Ion cell:

Nominal capacity, C (A.Sec) 7830

Modeling capacity, C1 (A.Sec) 1293.54

Modeling capacity, C2 (A.Sec) 12384.05

Modeling capacity, C3 (A.Sec) 4638.46

Modeliong resistance, R1 (Ohms) 0.00634

Modeliong resistance, R2 (Ohms) 0.00624

Modeliong resistance, R3 (Ohms) 0.00371

Cell Coulombic efficiency, Eta 1

Internal resistance for charging, R+ (Ohms) 0.03140

Internal resistance for discharging, R- (Ohms) 0.02550

Simulation parameters:

Sampling time (Sec) 0.062

Initial state estimation vector, xhat(0) [0 ; 0 ; 0 ; 85]

Matrix Q (n by n) 0.000001 0 0 ; 0 0 0.000001 0 ; 0 0 0 0.000001]

Matrix R (m by m) [5]

Matrix P0 (n by n) [0.1 0 0 0 ; 0 0.1 0 0 ; 0 0 0.1 0 ; 0 0 0 0.1]

Approximation of the open circuit voltage as a 10th-order polynomial function of SOC:

OCV = a10*SOC^10+a9*SOC^9+a8*SOC^8+a7*SOC^7+a6*SOC^6+a5*SOC^5+a4*SOC^4+a3*SOC^3+a2*SOC^2+a1*SOC+a0

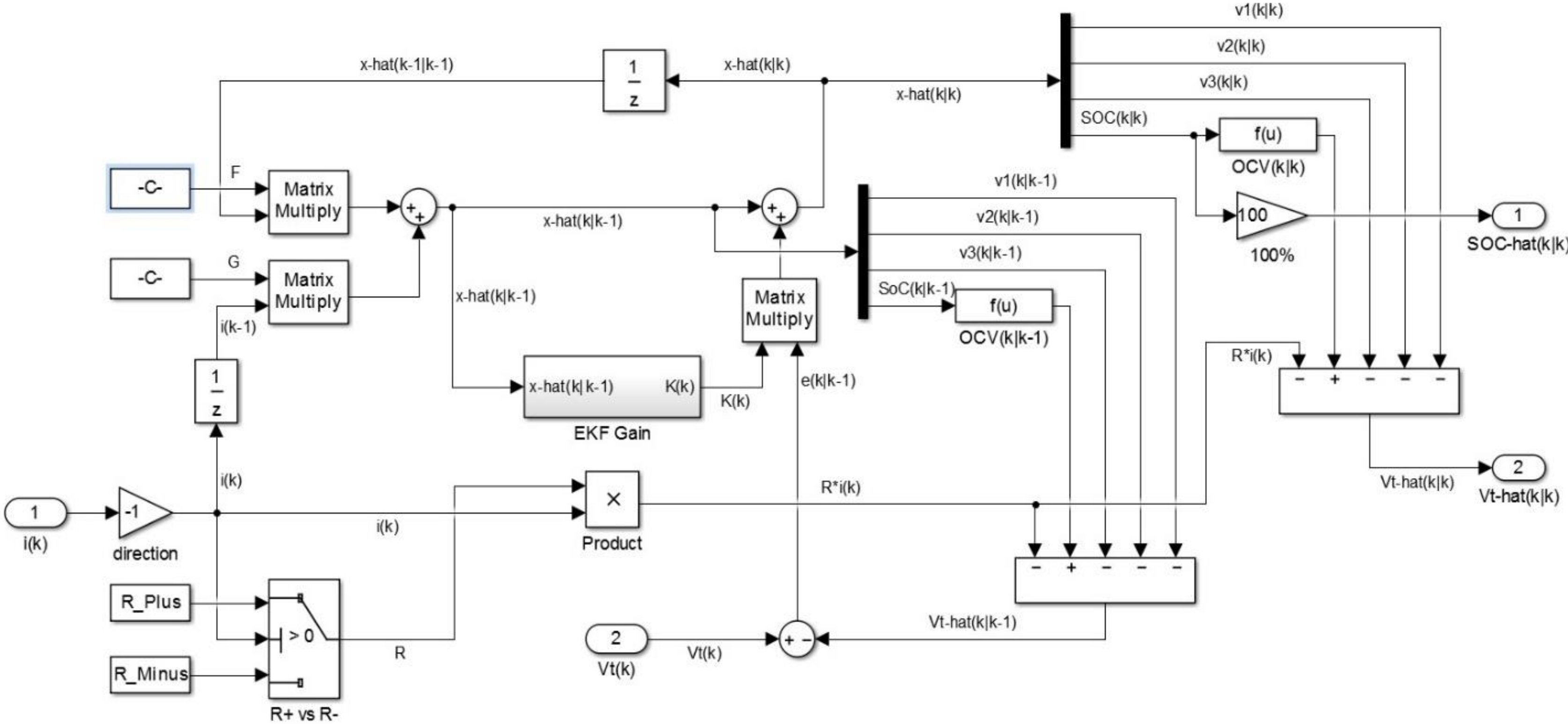
Enter the polynomial coefficients as: [a10 a9 a8 a7 a6 a5 a4 a3 a2 a1 a0]

998767932,-131706.2154282435,67987.9548086319,-22460.6473728408,4613.8659150179,-554.9927788049,35.6574029497,2.5292480288]

OK Cancel Help Apply

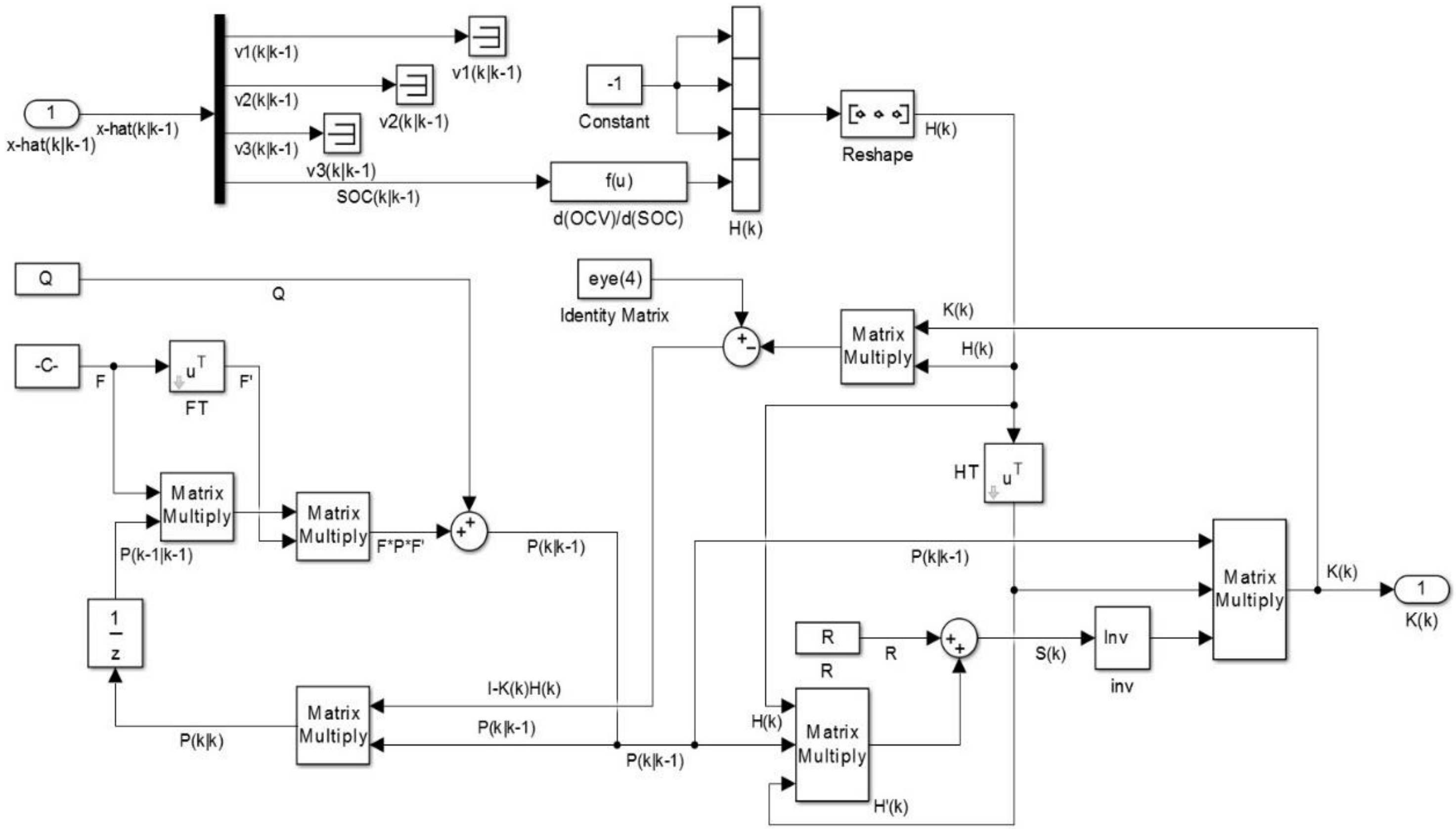
The EKF Model in Simulink

- A picture of the elements inside the EKF block



The EKF Gain in Simulink

- A picture of the elements inside the EKF gain:



SOC Estimation Using SVSF

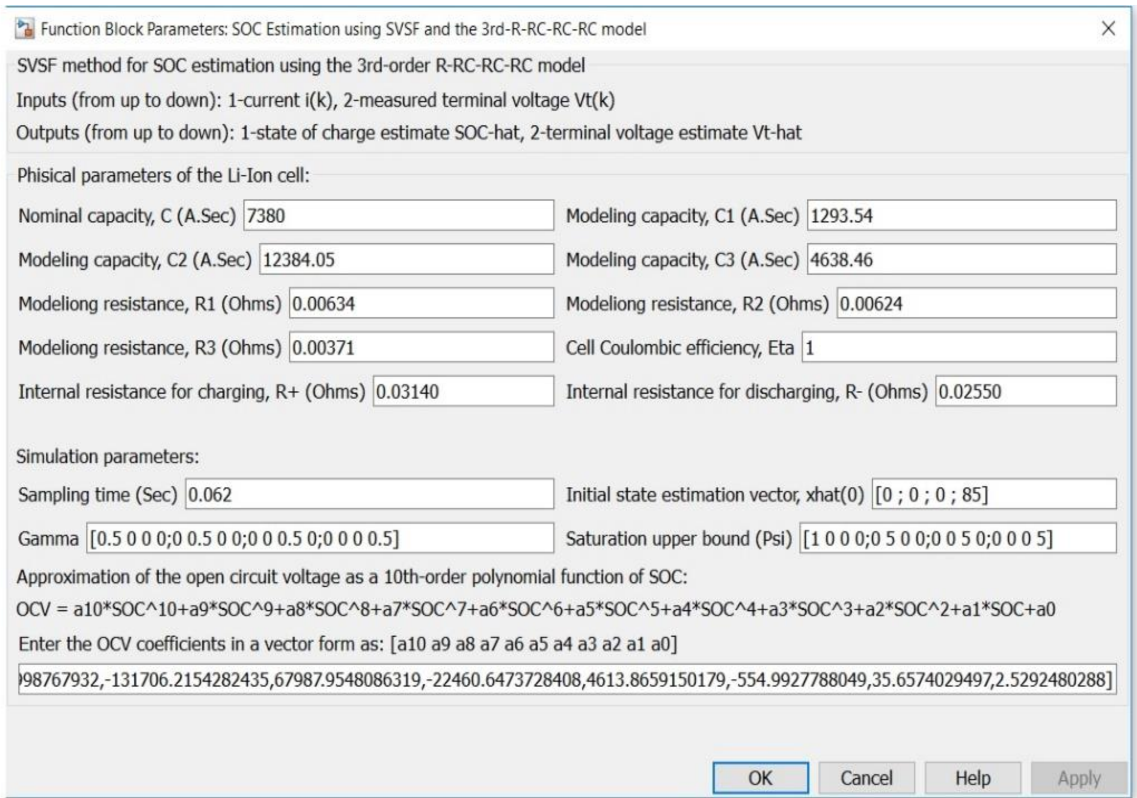
- For the SVSF method, the convergence rate matrix **γ** and the smoothing boundary layer width **ψ** are respectively set to:

$$\boldsymbol{\gamma} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}, \quad \boldsymbol{\psi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \quad \hat{\mathbf{x}}_0(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 85 \end{bmatrix}.$$

- Numeric values of **γ**, and **ψ** have been calculated by trial and error in order to achieve the best performance for the SVSF.
- Similar to EKF, the actual initial SOC is about 90.7%. $V_1(0)$, $V_2(0)$, $V_3(0)$ are not unknown and are assumed to be zero.
- The SVSF block has two inputs that are the current data “Current.mat” and the measured terminal voltage data “Voltage.mat”.

The SVSF User-Interface Window

- The SVSF user-interface window for input data:



Function Block Parameters: SOC Estimation using SVSF and the 3rd-R-RC-RC-RC model

SVSF method for SOC estimation using the 3rd-order R-RC-RC-RC model

Inputs (from up to down): 1-current i(k), 2-measured terminal voltage Vt(k)

Outputs (from up to down): 1-state of charge estimate SOC-hat, 2-terminal voltage estimate Vt-hat

Phisical parameters of the Li-Ion cell:

Nominal capacity, C (A.Sec) 7380

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Internal resistance for discharging, R- (Ohms) 0.02550

Simulation parameters:

Sampling time (Sec) 0.062

Initial state estimation vector, xhat(0) [0 ; 0 ; 0 ; 85]

Gamma [0.5 0 0 0;0 0.5 0 0;0 0 0.5 0;0 0 0 0.5]

Saturation upper bound (Psi) [1 0 0 0;0 5 0 0;0 0 5 0;0 0 0 5]

Approximation of the open circuit voltage as a 10th-order polynomial function of SOC:

OCV = a10*SOC^10+a9*SOC^9+a8*SOC^8+a7*SOC^7+a6*SOC^6+a5*SOC^5+a4*SOC^4+a3*SOC^3+a2*SOC^2+a1*SOC+a0

Enter the OCV coefficients in a vector form as: [a10 a9 a8 a7 a6 a5 a4 a3 a2 a1 a0]

98767932,-131706.2154282435,67987.9548086319,-22460.6473728408,4613.8659150179,-554.9927788049,35.6574029497,2.5292480288]

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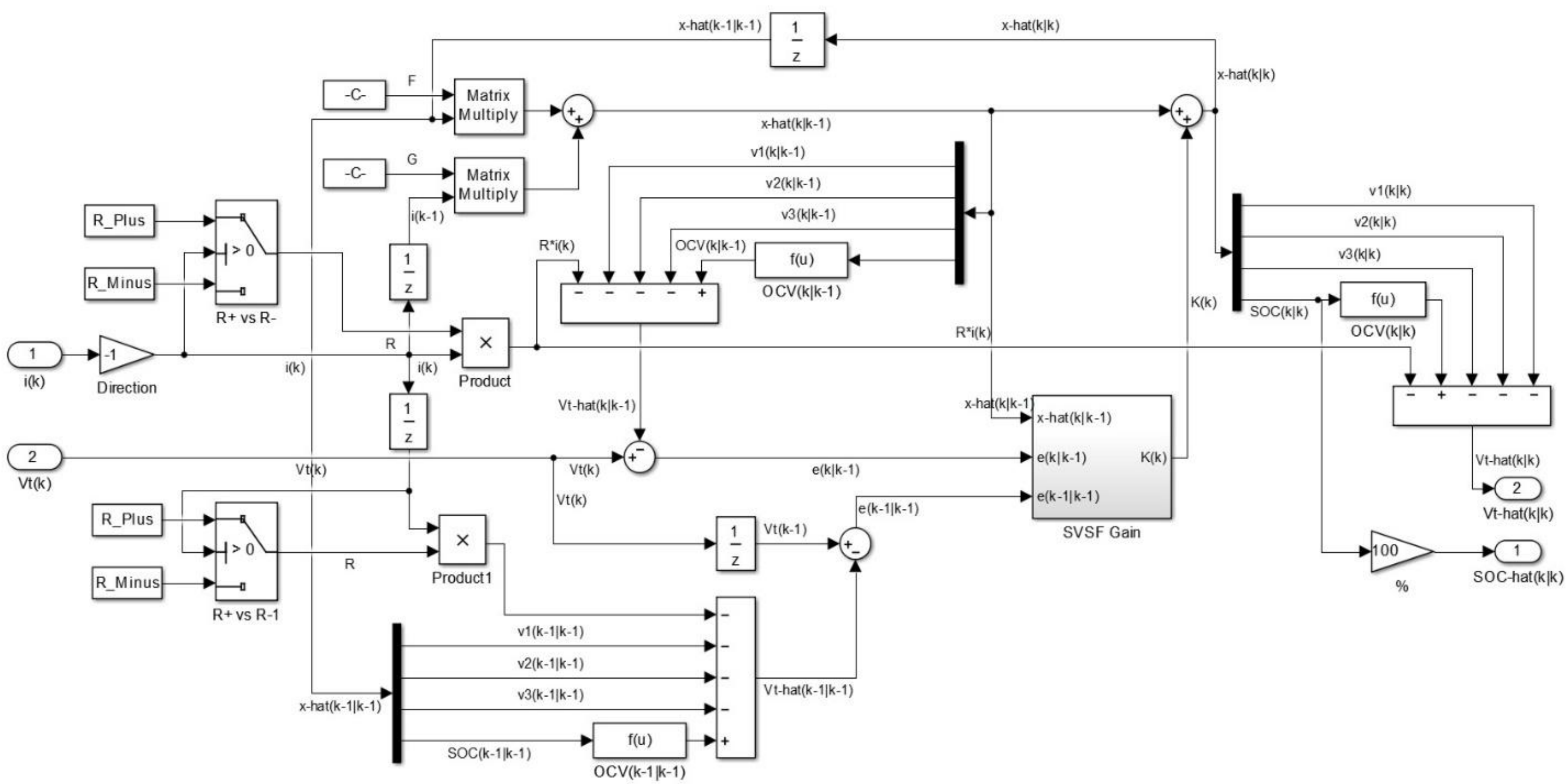
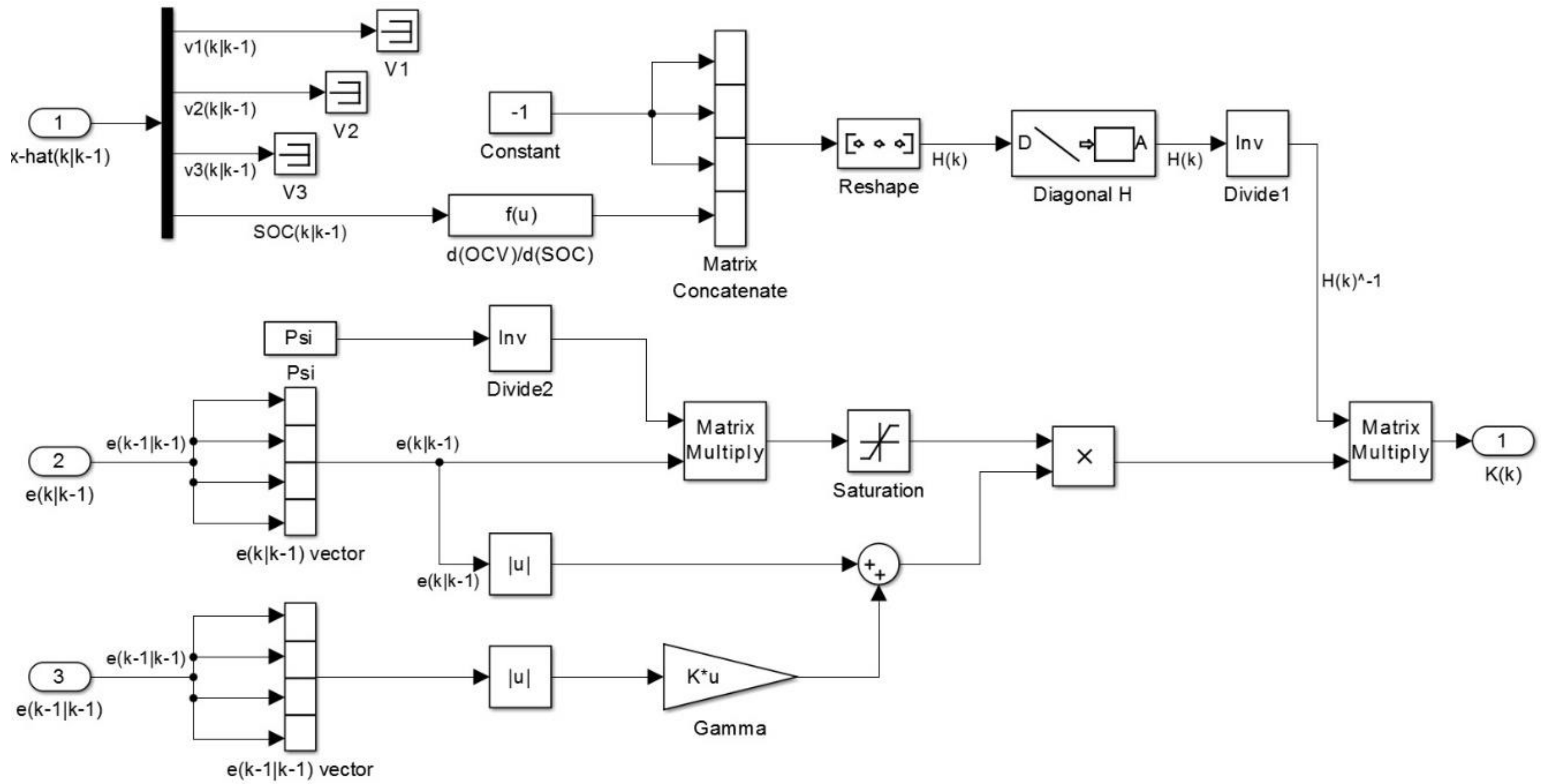
The SVSF Model in Simulink

- A picture of the elements inside the SVSF block:



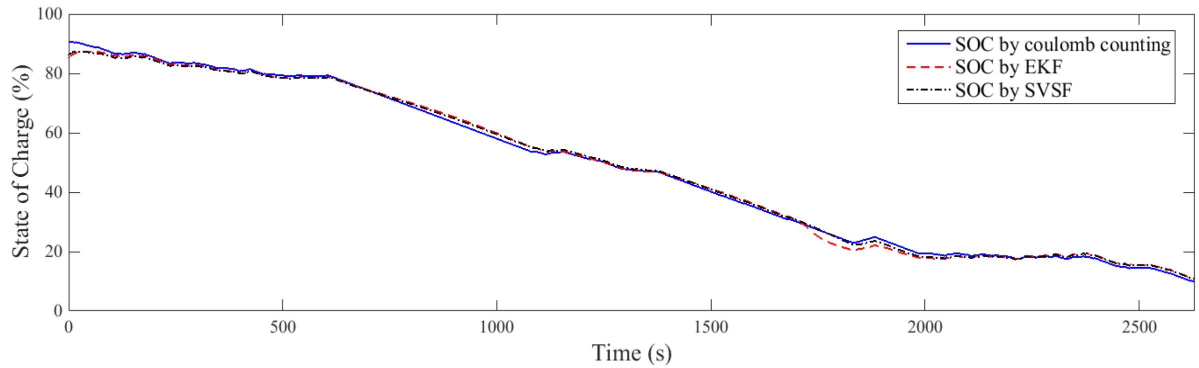
The SVSF Gain in Simulink

- A picture of the elements inside the SVSF gain:

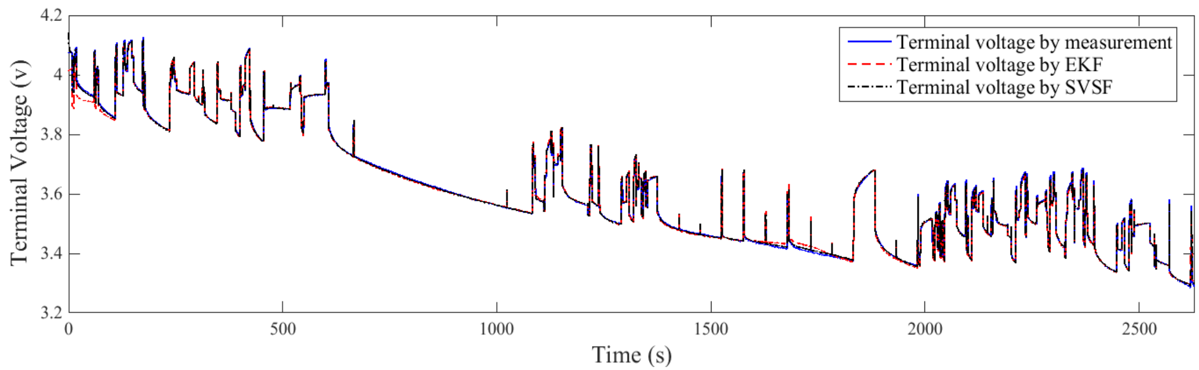


SOC Estimation Using 3rd-Order R-RC

- Using 3rd-R-RC model:
- Actual initial SOC=90.7% and initial SOC=85% :



- Vt estimate where initial SOC=85% :



	SOC (%)	Terminal Voltage (v)
EKF	2.935	0.0281
SVSF	2.842	0.0225

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Main Sources for Noise & Uncertainties

- Main sources for modeling uncertainties are:

1) uncertainties in the initial SOC,
2) inaccuracies by approximating cell dynamics via a circuit model,
3) averaging OCV-SOC curve for charge-discharge with a single curve,
4) parametrization error.

- Main sources for parametric uncertainties are:

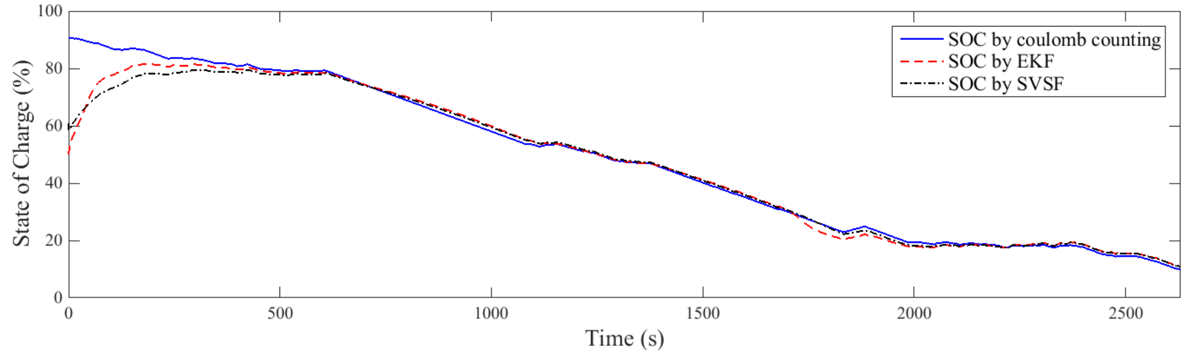
1) error in the parameter identification of the cell,
2) deviations of parameters from their nominal values due to aging.

- Main sources for noise are:

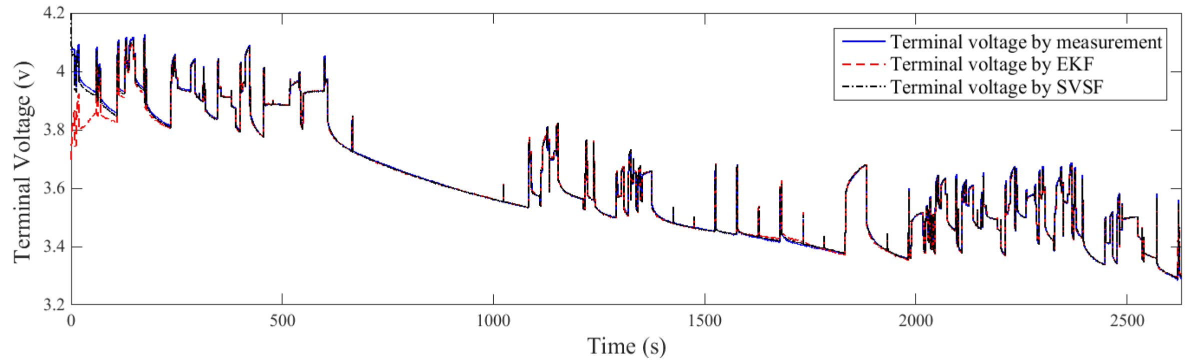
1) the instrumentation noise,
2) the voltmeter measurement noise,
3) unpredictable variations of the cell temperature.

Estimation with Uncertainties on Initial SOC

- Using 3rd-R-RC model:
- Actual initial SOC=90.7% and initial SOC=50% :



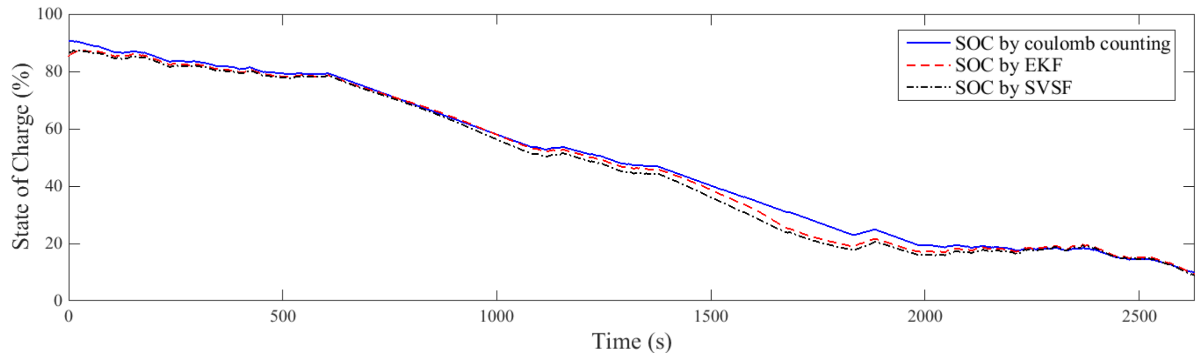
- Vt estimate where initial SOC=50% :



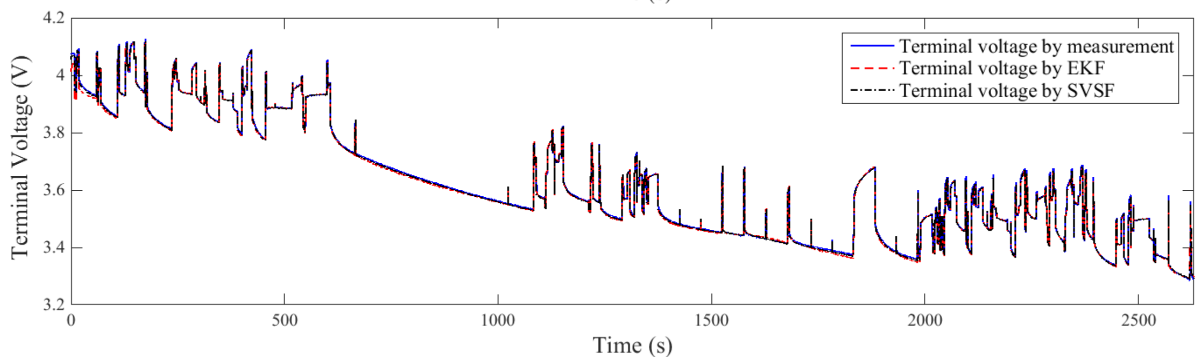
	SOC (%)	Terminal Voltage (v)
EKF	3.684	0.0278
SVSF	5.458	0.0229

Estimation with Parametric Uncertainties

- Using 3rd-R-RC model:
- SOC estimation where Actual C_n=75% of estimator's C_n:



- Vt estimation where Actual C_n=75% of estimator's C_n:



	SOC (%)	Terminal Voltage (v)
EKF	3.270	0.0318
SVSF	3.112	0.0314

Comparison of Robustness

- RMS values of the error for different levels of modeling uncertainty:

Modeling Uncertainty (%)	SOC (%)		Terminal Voltage (v)	
	SVSF	EKF	SVSF	EKF
0	0.969	0.893	0.0210	0.0228
10	1.618	1.757	0.0208	0.0239
20	2.999	3.166	0.0211	0.0276
30	4.604	4.704	0.0215	0.0332
40	6.442	6.457	0.0222	0.0400

- The table proves that the SVSF is more robust versus modeling uncertainties in comparison to the EKF.

Conclusion & Summary

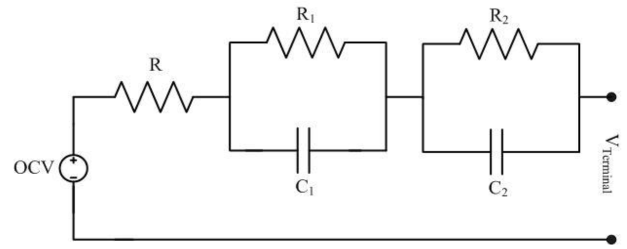
1) There are several sources for modeling and parametric uncertainties in the equivalent circuit modeling of Li-Ion cells.
2) The SVSF method benefits from the robustness characteristic of variable structure systems versus unknown uncertainties.
3) The SVSF produces more accurate SOC estimates over the EKF, where there are parametric uncertainties on. This is due to its robustness property versus modeling and parametric uncertainties.
4) The EKF provides more accurate SOC estimates over the SVSF, where there are uncertainties on the initial SOC estimate.

Assignment

- A 2nd-order R-RC model is defined as follows. Parameters of the model are listed in the table and the OCV-SOC relationship is similar to one used by the 3rd-order R-RC. Design a SOC estimators using the EKF and the SVSF method for the 2nd-order R-RC model in Simulink and compare the results.

$$\begin{bmatrix} V_{1,k+1} \\ V_{2,k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 & 0 \\ 0 & 1-\dfrac{\Delta t}{R_2 C_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} \\ \dfrac{\Delta t}{C_2} \\ -\dfrac{\eta \Delta t}{C} \end{bmatrix} i_k,$$

$$V_{\text{Terminal},k} = OCV(z_k) - V_{1,k} - V_{2,k} - Ri_k,$$



Numeric values of parameters for the R-RC-RC model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	28730.04 (Amp.s)
modeling resistance, R_1	0.00349 (Ohms)
modeling capacity, C_2	7583.62 (Amp.s)
modeling resistance, R_2	0.00664 (Ohms)
internal resistance, R_0^+	0.03731 (Ohms)
internal resistance, R_0^-	0.02564 (Ohms)
sampling time, Δt	0.062 (s)

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Main References

- H. Afshari, M. Attari, R. Ahmed, M. Farag, S. Habibi, “Modeling, parameterization, and state of charge estimation of Li-Ion cells using a circuit model”, IEEE Transportation Electrification Conference and Expo (ITEC), 2016.
- H. H. Afshari, S. A. Gadsden and S. H. Habibi, "Gaussian filters for parameter and state estimation: A general review of theory and recent trends," *Signal Processing,* vol. 135, p. 218–238, 2017.
- S. Habibi, "Smooth variable structure filter," *Proceedings of the IEEE,* vol. 95, no. 5, pp. 1026-1059, 2007.
- G. L. Plett, "Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs: part 2: modeling and identification", *Journal of Power Sources,* vol. 134, no. 2, pp. 262-276, 2004.
- M. S. Farag, R. Ahmed, S. A. Gadsden, S. R. Habibi and J. Tjong, "A comparative study of Li-ion battery models and nonlinear dual estimation strategies," in *IEEE Transportation Electrification Conference and Expo*, Dearborn, USA, 2012.

Any Question?

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