McMaster University ENGINEERING

MODELING LI-ION CELLS USING EQUIVALENT CIRCUITS IN MATLAB-SIMULINK

CMHT
CENTRE FOR MECHATRONICS AND HYBRID TECHNOLOGIES

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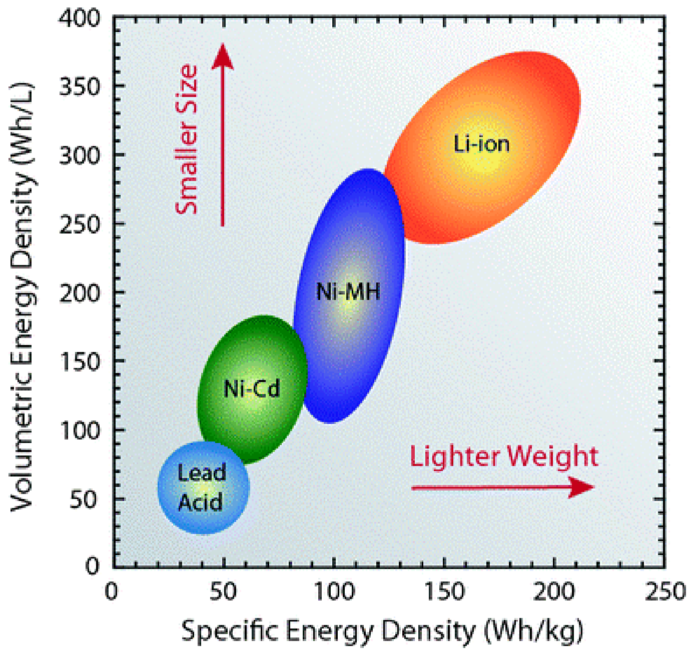
Contents

- Li-Ion batteries
- Test setup and test procedures
- The 1st-order R-RC Model
- Parameterization via the genetic algorithm
- Introduction to MATLAB-Simulink
- Modeling via Simulink

Why Using Li-Ion Cells

Advantages of Li-Ion cells

- Higher energy density
- Slow loss of charge when disconnected
- Smaller hysteresis effect
- Durability



Disadvantages of Li-Ion cells

- Tendency to overheat
- Damage at high voltages

http://www.cei.washington.edu/education/science-of-solar/battery-technology/

Cell, Module, and Pack

- **Cell:** The smallest unit connected in parallel or series to make a module
- **Pack:** A module is connected in parallel or series to make a pack. Voltage, current, and temperature are measured by sensors from each module.

The battery cell, module (4 cells), and pack (48 modules) used by Nissan Leaf

http://www.nissan-global.com/EN/TECHNOLOGY/MAGAZINE/ev_battery.html

Battery Definitions

C-Rate: The rate at which the battery is charged or discharged with respect to its maximum capacity (1 C-rate is the current that discharges the cell in 1 hour)

Open-Circuit Voltage: The voltage across the cell terminals when no load is applied.

Terminal Voltage: The voltage across the cell terminals when a load is applied.

Nominal Capacity: The total Amp-hour that can be drawn from a healthy battery when the battery is discharged at a certain C-rate.

Internal Resistance: The resistance within the battery components that converts the charging energy into heat.

http://web.mit.edu/evt/summary_battery_specifications.pdf

Battery Conditions

Cycle Life: The number of full charging/discharging cycles before the cell's capacity drops to 80% of its initial capacity. It is a function of the operating temperature and the C-rate.

State of Charge (SOC): The battery's current capacity as a percentage of its maximum capacity. The SOC is not directly measured by any sensor. It can be calculated using the current integration or can be obtained by state estimation.

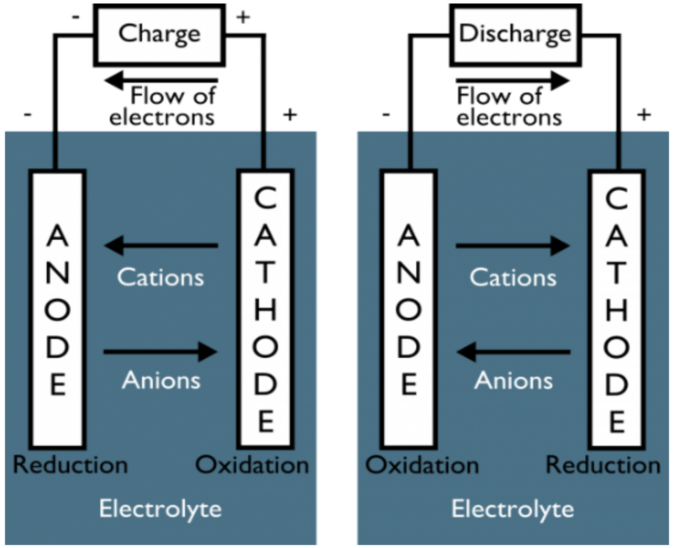
Depth of Discharge (DOD): The battery capacity that is discharged as a percentage of its maximum capacity.

State of Health (SOH): It is an indicator that reflects the general condition of a battery (a cell or a pack) compared to its ideal condition.

http://web.mit.edu/evt/summary_battery_specifications.pdf

Main Elements of a Cell

- Main elements of a battery include: anode, cathode, electrolyte, separator, and current collectors.
- Anode is the source of reduction reactions during charge and oxidation reactions during discharge. Cathode is the source of oxidation reactions during charge and reduction reactions during discharge.



http://storage4.eu/2013/03/rechargeable-battery-diagram-convention/

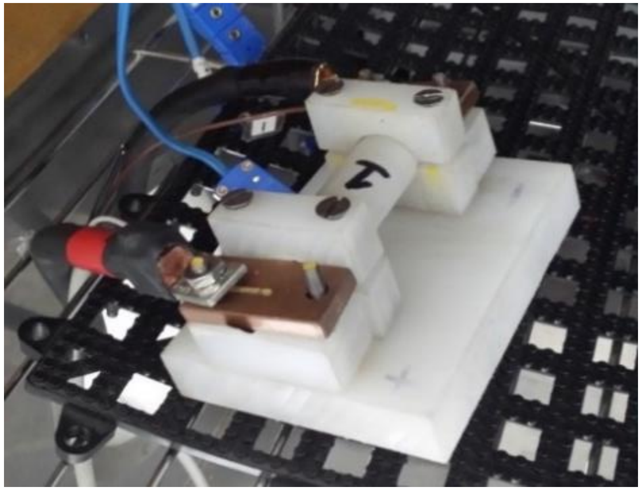
Charge-Discharge Process

- **Discharge:** During discharge, the anode is oxidized and the cathode is reduced. This causes that electrons move from anode (-) to cathode (+). As the anode looses electrons, it attracts anions. Otherwise, the cathode attracts electrons, and the cell starts loosing the charge.
- **Charge:** During charge, a voltage applies to the cell and it causes that electrons move in the other direction (note that (-) and (+) stay at the same sides, since electrons flow in the direction they left from before). Even though, the left electrode is now reduced and the right electrode is oxidized, for the convention, anode's and cathode's name stay the same. But, from a scientific point of view, the cathode and anode will depend on the state of the battery (charge vs. discharge).

http://storage4.eu/2013/03/rechargeable-battery-diagram-convention/

A Typical Battery Test Setup

- Main elements of a test setup: 1-environmental chamber, 2-power supply, 3-data acquisition card, 4-current sensor, 5-thermocouples, and 6-safety circuit:

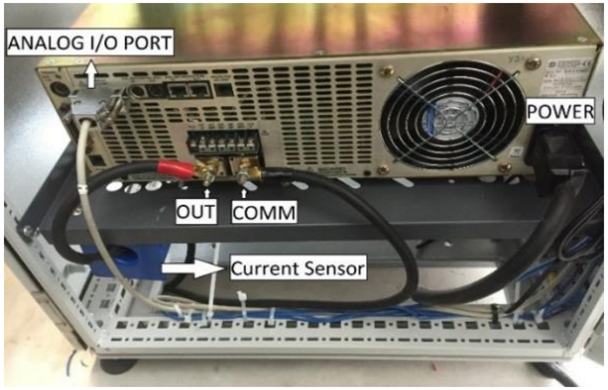


A Li-Ion cell inside the holder

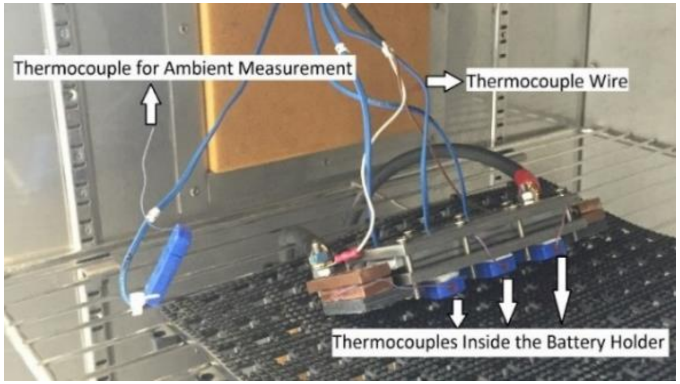
The battery test setup

Elements of a Battery Testbed

- The environmental chamber is used to vary the temperature between -66°C to 177°C.
- The power supply provides current for the Li-Ion cell in the range of ±6V and ±150 A. It is controlled by a computer using the data acquisition card.
- The setup uses three thermocouples to measure the cell temperature and another one to measure the ambient temperature.



The power supply



The four thermocouples

Three Main Test Scenarios

Three tests are mainly run to model a Li-Ion cell:

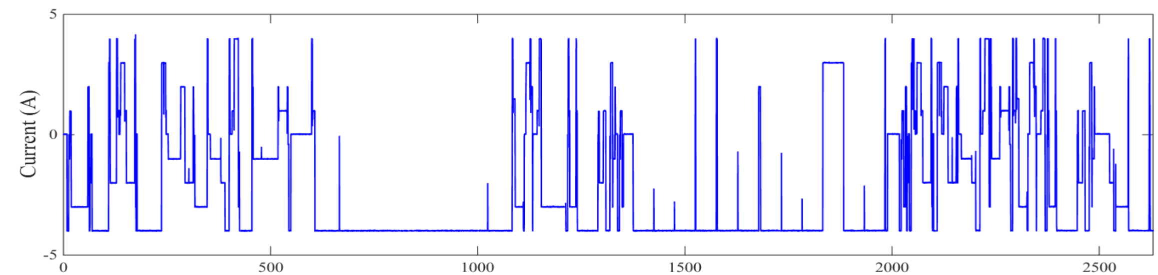
1) **The static capacity test:** To measure the cell's nominal capacity C_n at a constant-current discharge rate with 1 C-rate;

2) **The SOC-OCV test:** To obtain the SOC-OCV relationship by charging and discharging at 1/15 C-rate. The averaged curve for charge and discharge is approximated by a high-order polynomial function;

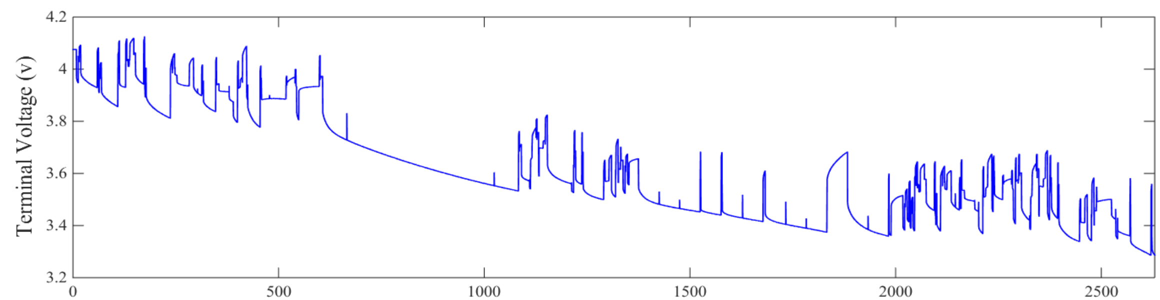
3) **The cycle tests**: To excite battery's dynamics and measure input-output (current-terminal voltage) data that are later used to identify parameters of the equivalent circuit model.

Sample of Cycling Tests

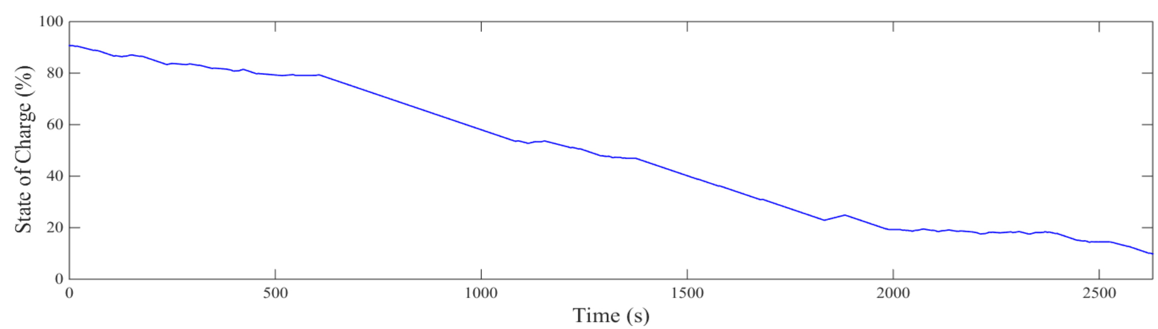
- Input current cycle:



- Measured terminal voltage:



- Calculated SOC (Coulomb-counting):



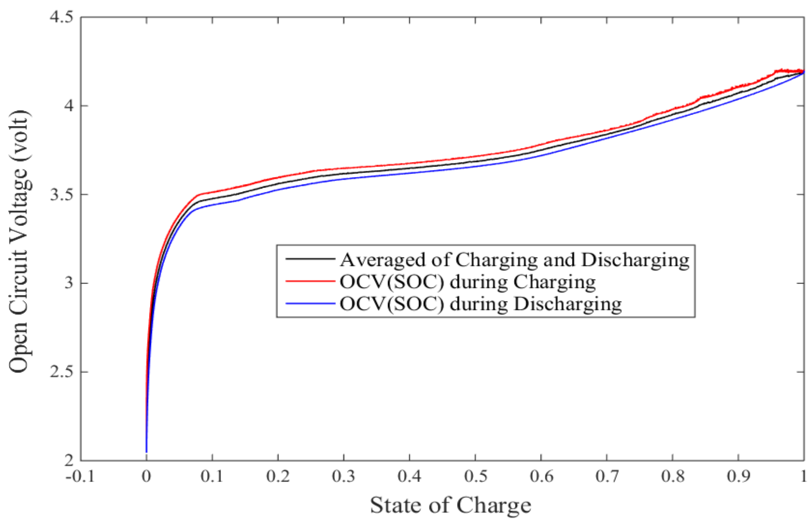
OCV-SOC Curve Derivation

- The OCV-SOC curve is obtained by taking the average of the OCV-SOC curve for charge and discharge with a C/15 rate. This curve is then approximated using a high-order polynomial function.

$$OCV(z_k) = p_{10}z_k^{10} + p_9z_k^{9} + p_8z_k^{8} + p_7z_k^{7} + p_6z_k^{6} + p_5z_k^{5} + p_4z_k^{4} + p_3z_k^{3} + p_2z_k^{2} + p_1z_k + p_0.$$

COEFFICIENTS OF THE 10TH-ORDER SOC-OCV POLYNOMIAL

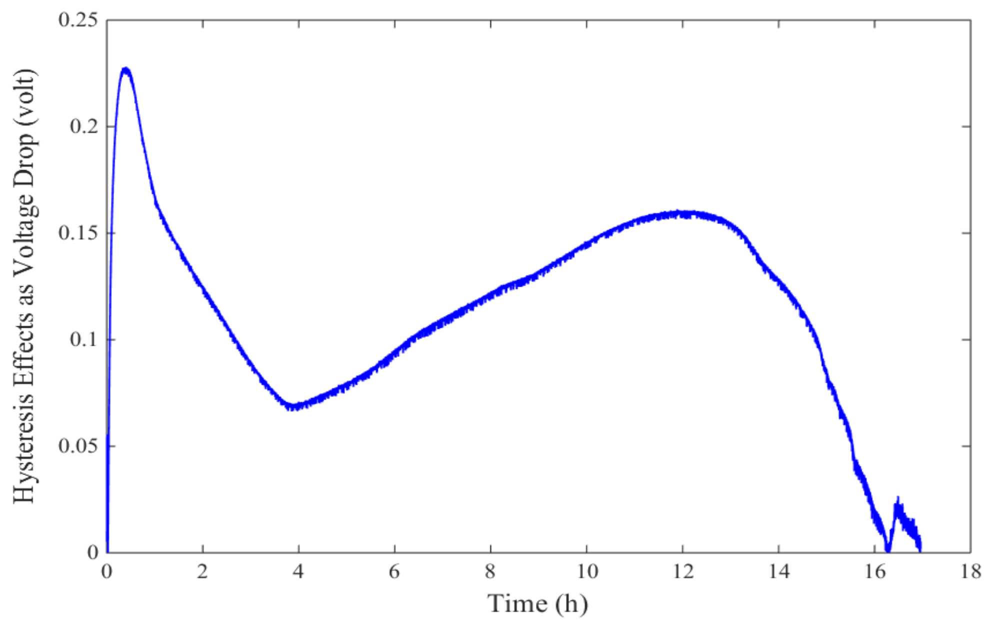
Coefficient	p_1	p_2	p_3	p_4
Numeric value	-10150.68	54373.42	-125525.42	163388.70
Coefficient	p_5	p_6	p_7	p_8
	-131706.22	67987.96	-22460.65	4613.87
Coefficient	p_9	p_{10}	p_{11}	
Numeric value	-554.99	35.66	2.53	



Profiles for charge, discharge and the averaged one

Hysteresis Curve Derivation

- The hysteresis is equal to the amount of the voltage drop during charging and discharging. It is determined by subtracting the SOC-OCV curve for discharging from the SOC-OCV curve for charging.



Profile of the hysteresis drop in voltage

Different Modeling Techniques

- The choice among modeling approaches is a compromise between **model complexity**, **accuracy**, and **computational cost**:

Increasing Complexity & Accuracy

Empirical Modeling

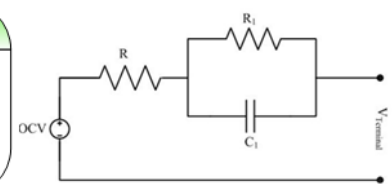
- Represent hysteresis, polarization time, Ohmic loss
- No physical significance, less parameters to tune
- Applicable for SOC estimation but not for SOH estimation
- Lower accuracy of modeling

$$z_{k+1} = z_k - \left(\frac{\eta_i \Delta t}{C_n}\right) i_k,$$

$$y_k = K_0 - R i_k - \frac{K_1}{z_k} - K_2 z_k + K_3 \ln(z_k) + K_4 \ln(1 - z_k).$$

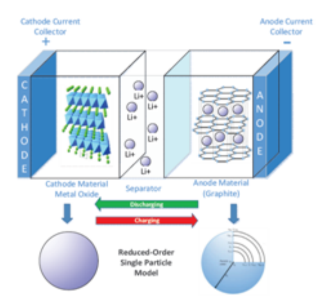
Equivalent Circuit Modeling

- Represent hysteresis, Ohmic loss, different polarization constants
- Average physical significance, less parameters to tune
- Applicable for SOC estimation but not for SOH estimation
- Easy and accurate for implementation and computationally efficient



Electro-chemical Modeling

- Represent Lithium diffusion
- High physical significance, more parameters to tune
- Applicable for SOC and SOH estimation
- Difficult to obtain parameters and computationally expensive



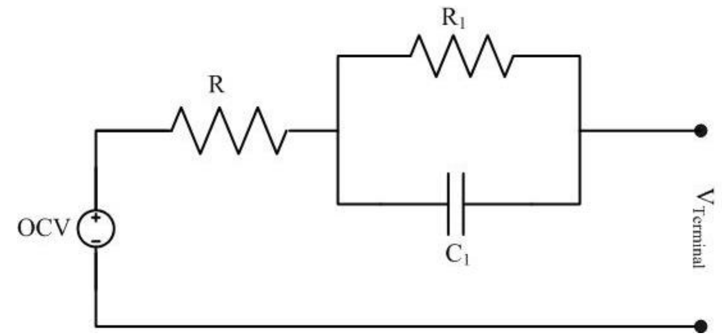
Increasing Simplicity with Smaller Physical Significance

The 1st-Order R-RC Model

- A 1st-order R-RC model in state-space:

$$\begin{bmatrix} V_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_k \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} \\ -\dfrac{\eta_i \Delta t}{C} \end{bmatrix} i_k,$$

$$V_{\text{Terminal},k} = OCV(z_k) - Ri_k - V_k,$$



Δt is sample time, *C* is nominal capacity, and *η* is columbic efficiency.

- The 1st-order R-RC model consists of an internal resistance *R*, a modeling resistance R_1, and a modeling capacitance C_1. Value of *R* is a function of the current direction. It is *R+* for charging and *R-* for discharging.

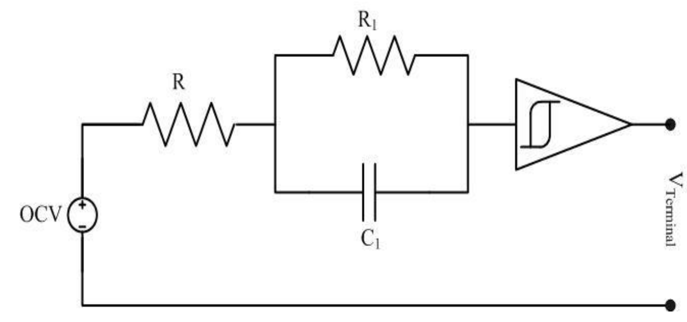
- There are two state variables including *V* the voltage across C_1, and *z* the state of charge. The input to the model is the current i_k, and the output is $V_{Terminal}$ that is the terminal voltage across the two ends of the cell.

Parameters of the 1st-Order R-R-C

- The equivalent circuit is a grey-box modeling method in which the model structure is known and parameters are unknown. The structure is derived from circuit equations and parameters are obtained using optimization.
- The OCV denotes the open-circuit voltage variable defined as a polynomial function of z_k. This function is obtained using the OCV-SOC characterization test that is explained later.
- Value of C *is* obtained by the static capacity test, η is assumed to be equal to one, and value of Δt depends on the sampling rate of the voltmeter.
- Values of C_1, R_1, $R+$, $R-$ are obtained by capturing input-output data and applying genetic algorithm for parameterization. It minimizes the error that is the difference between the simulated and measured terminal voltage.

The 1st-Order R-RC-H Model

- The R-RC-H model uses a R-C and a hysteresis element to represent the cell dynamics:



$$\begin{bmatrix} V_{1,k+1} \\ h_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 & 0 \\ 0 & F(i_k) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ h_k \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} & 0 \\ 0 & 1-F(i_k) \\ -\dfrac{\eta \Delta t}{C} & 0 \end{bmatrix} \begin{bmatrix} i_k \\ M(\mathrm{z}, \dot{\mathrm{z}}) \end{bmatrix},$$

$$V_{\mathrm{Terminal},k} = OCV(z_k) - V_k + h_k - R i_k,$$

- In addition to *C*, *C1*, *R1*, *R+*, *R-*, *OCV*, the 1st-order R-RC-H uses a hysteresis element *h* to represent the hysteresis effect. The hysteresis is the difference between the cell's voltage on charge and discharge at the same SOC value.
- The hysteresis effect is quantified comparing the SOC-OCV characterization tests for charging and discharging.

Parameters of the 1^{st}-Order R-RC-H

- The R-RC-H model has three states including *V*, *z*, and the hysteresis state *h*. It has two inputs including i_k, and M_k, which is the polarization constant and it has two values including *M+* for charging and *M-* for discharging. The output $V_{Terminal}$ is a function of *OCV*, *V*, *R×i*, and *h*.

- The hysteresis effect is calculated using the following function:

$$F(i_k) = \exp\left(-\left|\frac{\eta\gamma\Delta t i_k}{C}\right|\right),$$

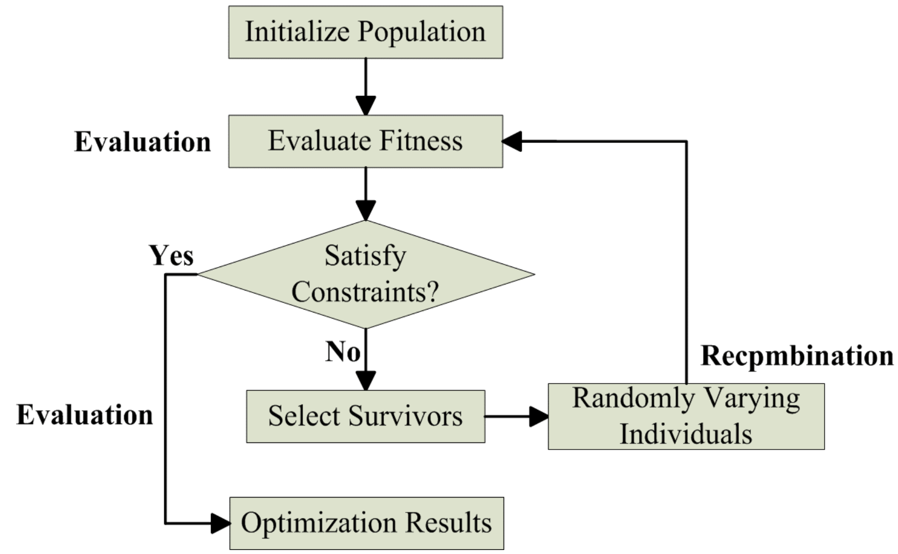
where Υ is the hysteresis rate, and η is the columbic efficiency.

- Values of C_1, R_1, *R+*, *R-*, *M+*, *M-* and Υ are obtained by capturing input-output data and applying the genetic algorithm for optimization. Note that upper bounds for *M+*, *M-* and Υ are set based on the hysteresis curve.

Genetic Algorithm for Optimization

- Genetic algorithm is a class of stochastic search strategies that are modeled by evolutionary mechanisms.
- They can optimize nonlinear systems with a large number of variables.
- Main steps of a genetic algorithm:

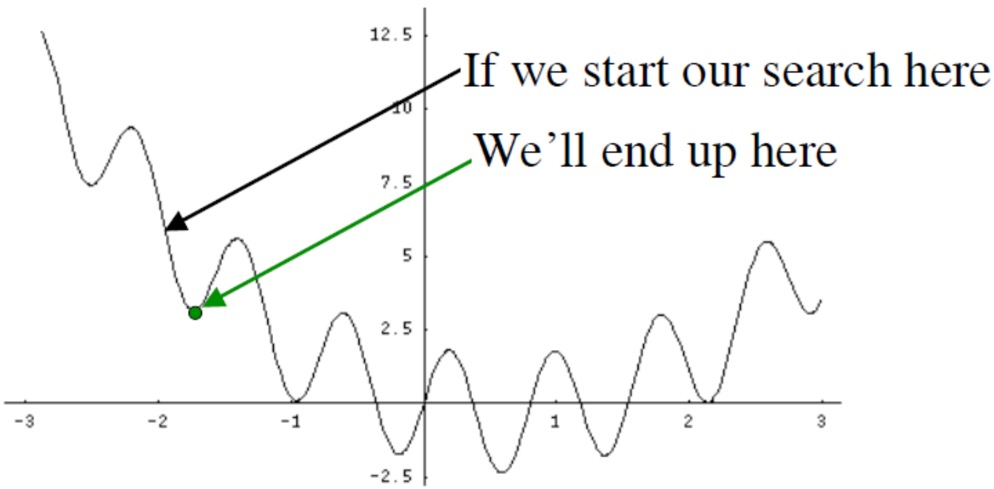
1) Choose parameters to optimize
2) Create initial population of individuals
3) Evaluate fitness of each individual
4) Apply selection rules and random behavior to select survivors
5) Create new individuals randomly



https://www.msi.umn.edu/sites/default/files/OptimizingWithGA.pdf

Genetic Algorithm vs Newton-Raphson

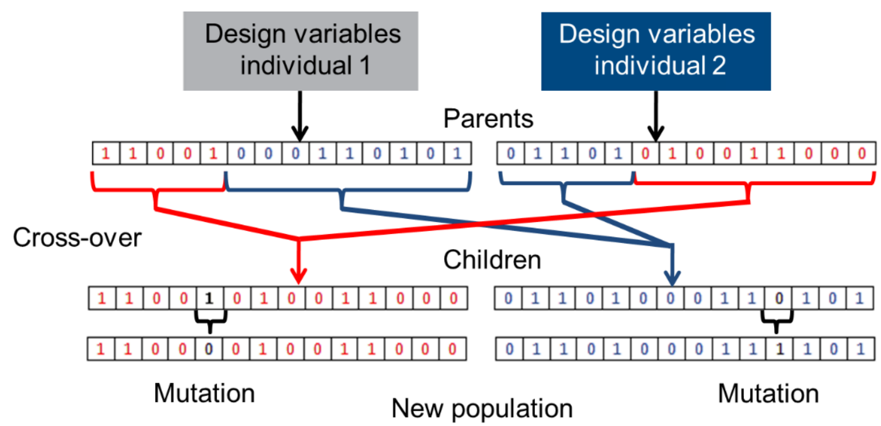
- Newton-Raphson and its variants use local information for optimization.
- They use the function value and its derivative with respect to the optimized parameters to find the local maximum or minimum.
- The Newton-Raphson approach fails, since a local method can only find local extrema.



https://www.msi.umn.edu/sites/default/files/OptimizingWithGA.pdf

Genetic Algorithm for Parameterization

- The model is obtained from the equivalent circuit (e.g. 1st-order R-RC), where its parameters are unknown. The three test procedures are run. Excitation tests produce input-output (current-terminal voltage) data.
- The genetic algorithm applies using the 1st-order R-RC model and input-output data to identify parameters of the model. It minimizes the error that is the difference between the simulated and the measured output.
- For the case study, the population size and the generation number are respectively set to 200, and 15. For more accurate results, these numbers need to be larger.



http://www.mdpi.com/1996-1073/9/3/181/htm

Parameterization Results

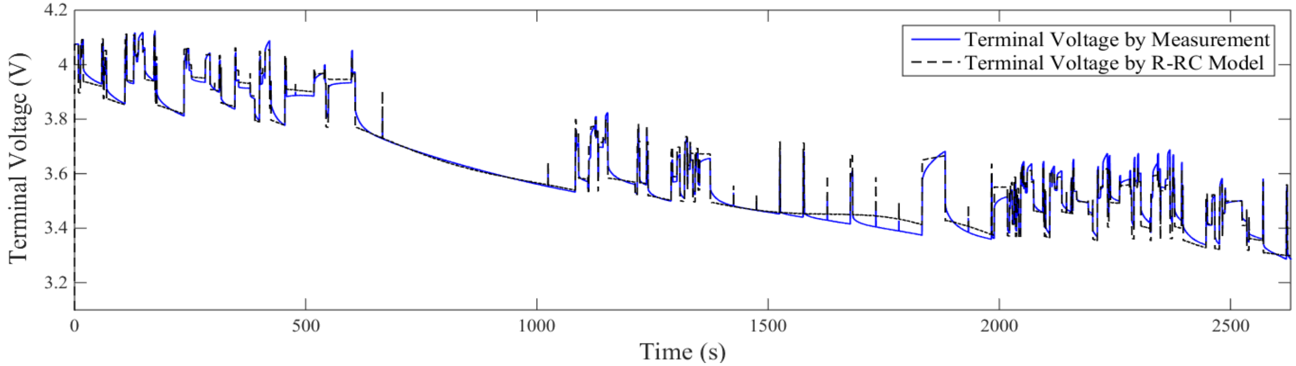
- Parameters of the 1st-order R-RC model with and without hysteresis are:

Numeric values of parameters for the R-RC model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	6618.4 (Amp.s)
modeling resistance, R_1	8.62e-05 (Ohms)
internal resistance, R_0^+	0.0445 (Ohms)
internal resistance, R_0^-	0.0217 (Ohms)
sampling time, Δt	0.062 (s)

Numeric values of parameters for the R-RC-H model

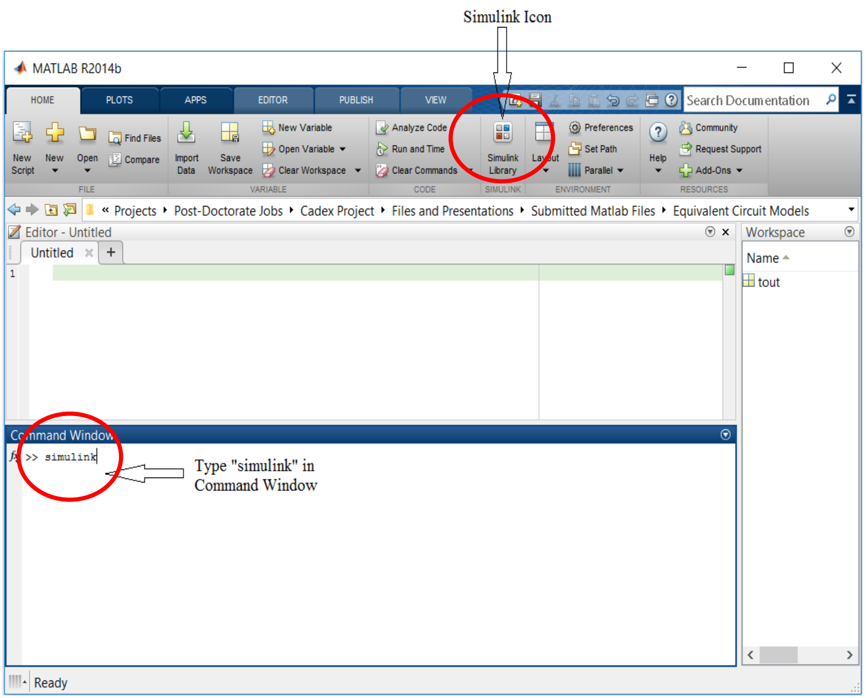
Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	2775.92 (Amp.s)
modeling resistance, R_1	0.0172 (Ohms)
internal resistance, R_0^+	0.0360 (Ohms)
internal resistance, R_0^-	0.0248 (Ohms)
max polarization constant, M^+	0.0178
max polarization constant, M^-	5.384e-05
hysteresis rate, γ	16.676

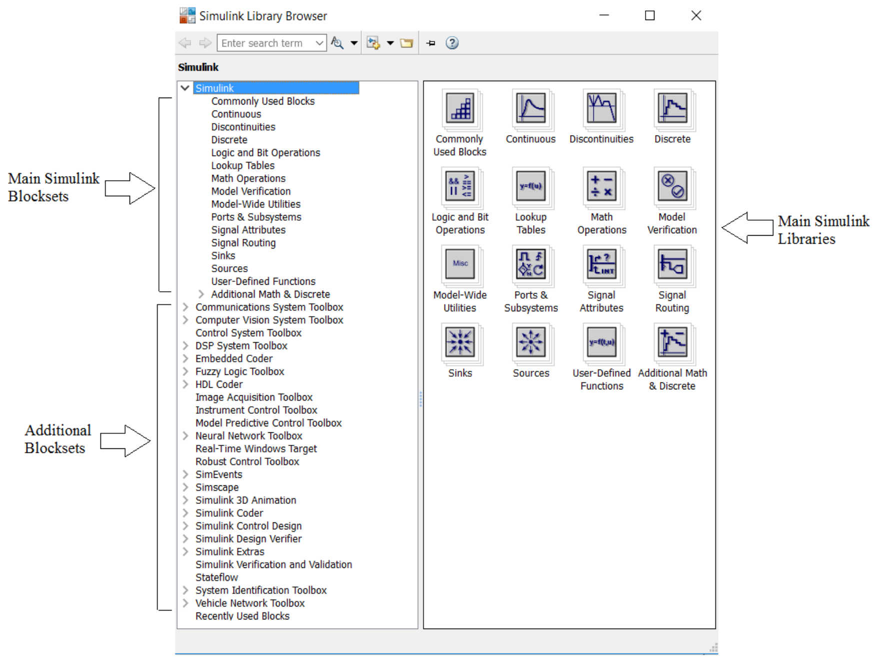


Comparison between the simulated and the measured terminal voltage

Introduction to MATLAB-Simulink

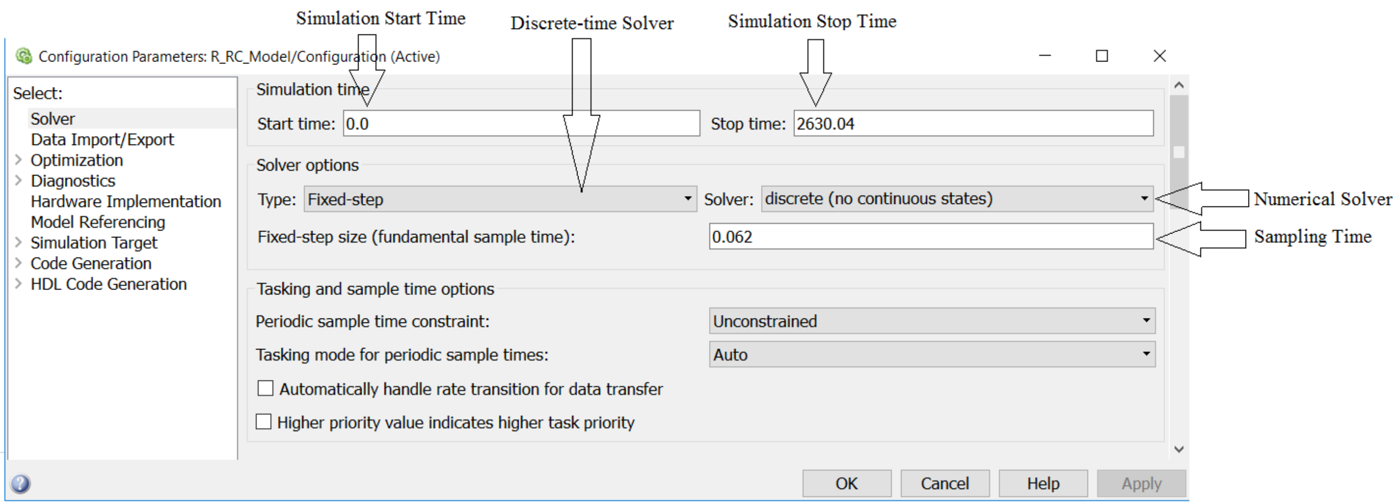
- How to run a Simulink file:

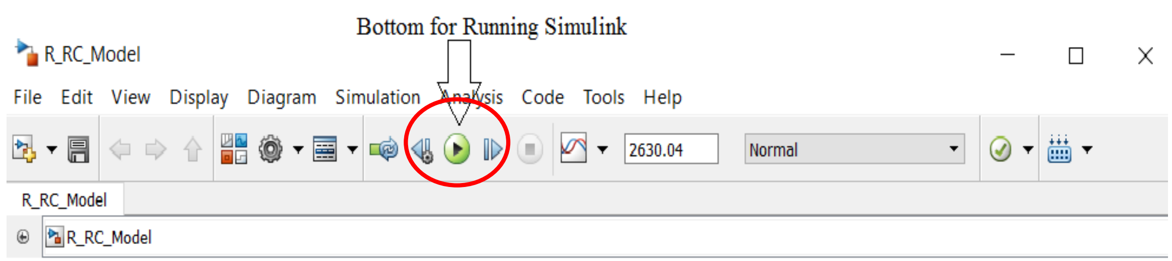




How to Run Simulink

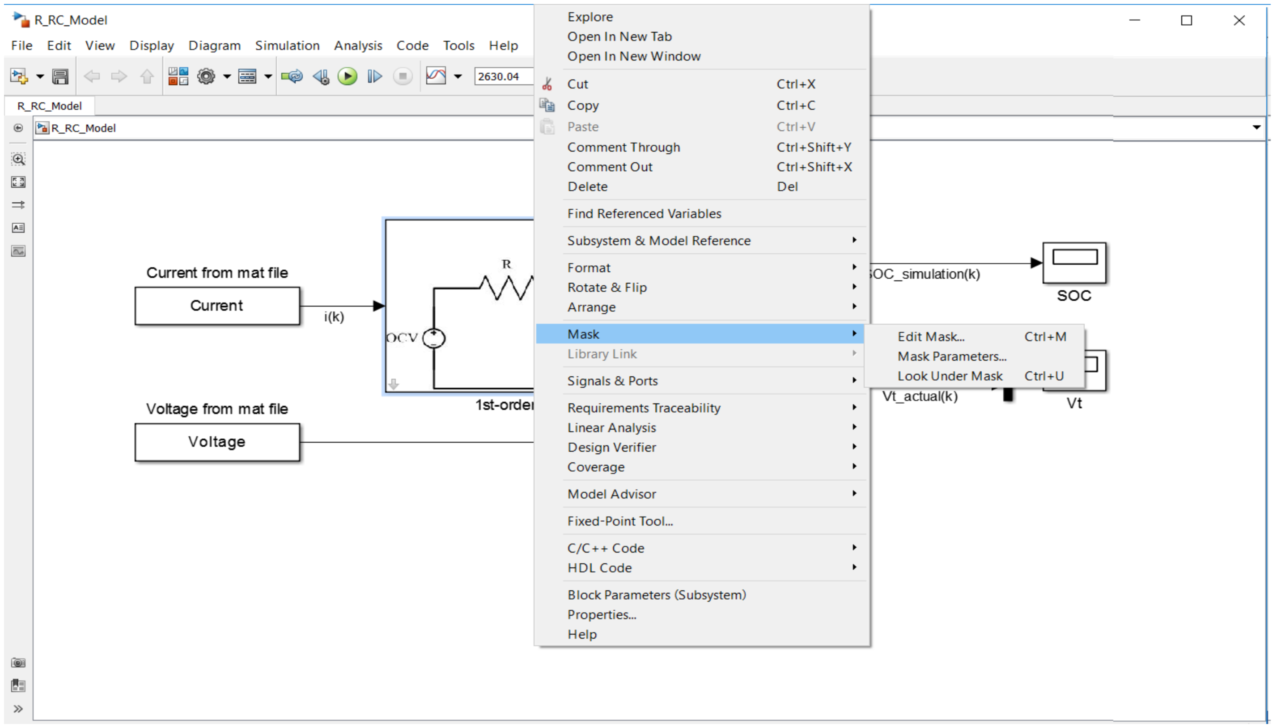
- How to adjust the numeric solver:





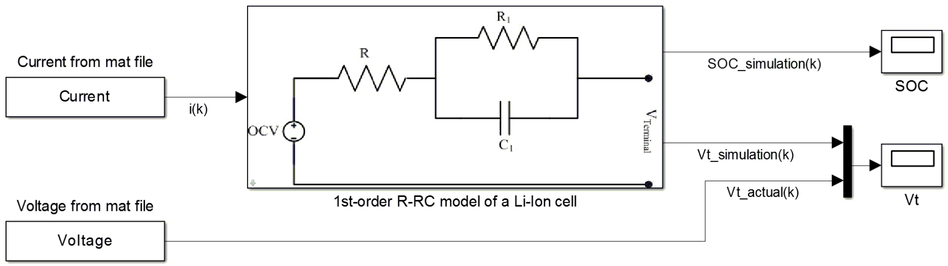
MASK Interface

- How to create a mask interface on the model
- Right click on the model to see the mask options as:

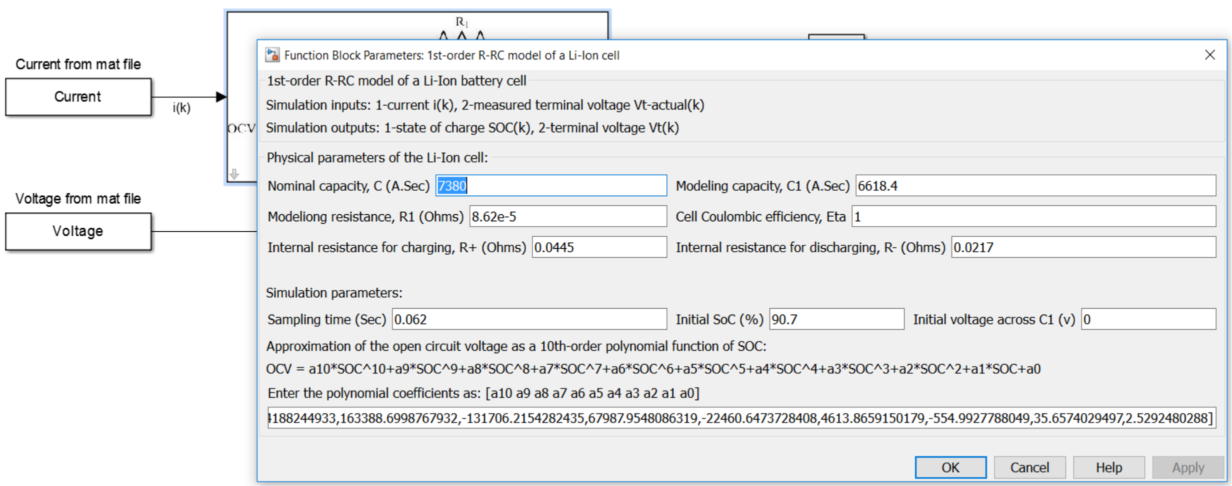


1st-Order R-RC Model in Simulink

- How to feed parameters into the 1st-R-RC model:



- Double click on the model to see:



How to Run the Files

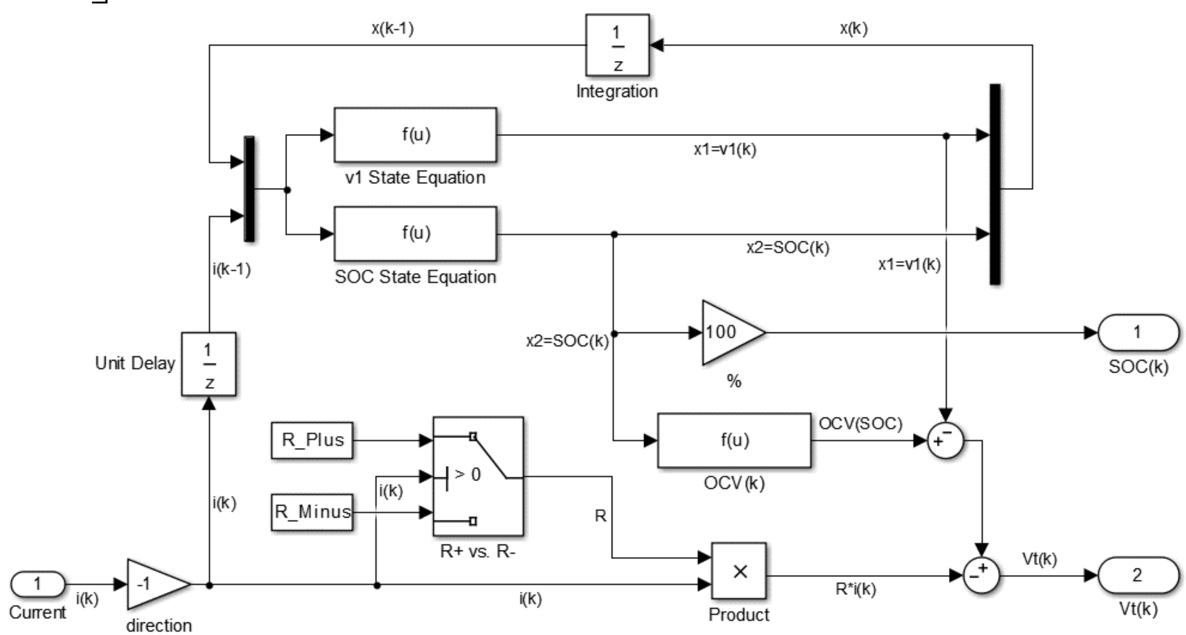
- There are two Simulink files: R_RC_Model.slx and R_RC_H_Model.slx. They respectively represent dynamics of a Li-Ion cell using a 1st-order R-RC model without and with the hysteresis element.
- Supporting files include two JPEG files as cover photos for models, and two MATLAB files named by Current.mat, and Voltage.mat. They are linked to model and contain the input current and the measured terminal voltage.
- Simulink files can only be run using MATLAB R2014b and probably newer versions. Parametric values for these models are fed into models using a mask interface built in Simulink models.
- The Simulink solver is set to discrete that is a fixed-step solver. The sample time for test and simulations is equal to 0.062 sec and the total running period is 2630.04 sec.

Look into the 1st-Order R-RC Model

- How to create the Simulink model using blocks:

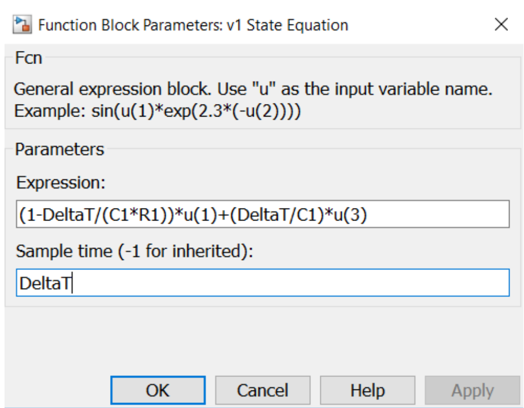
$$\begin{bmatrix} V_{1,k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} \\ -\dfrac{\eta \Delta t}{C} \end{bmatrix} i_k,$$

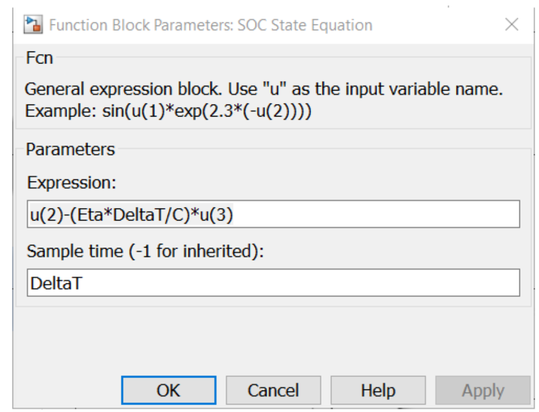
$$V_{\mathrm{Terminal},k} = OCV(z_k) - V_{1,k} - Ri_k,$$



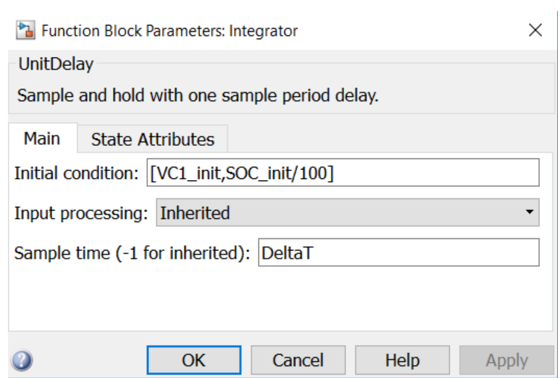
State Equation Blocks

- How to add the state equation blocks into Simulink:



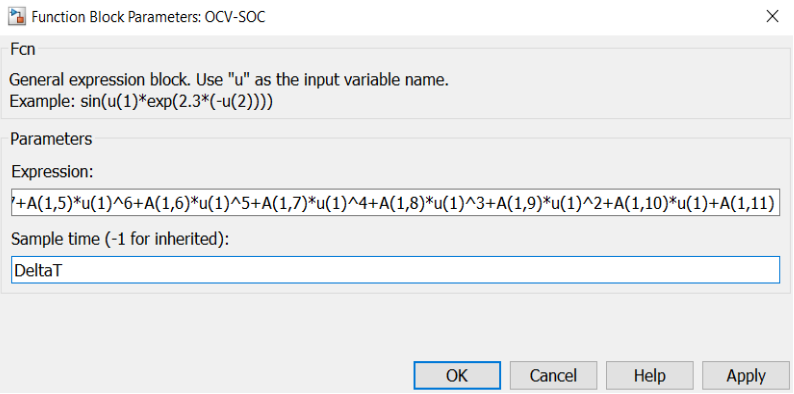


- How to initiate the integration:

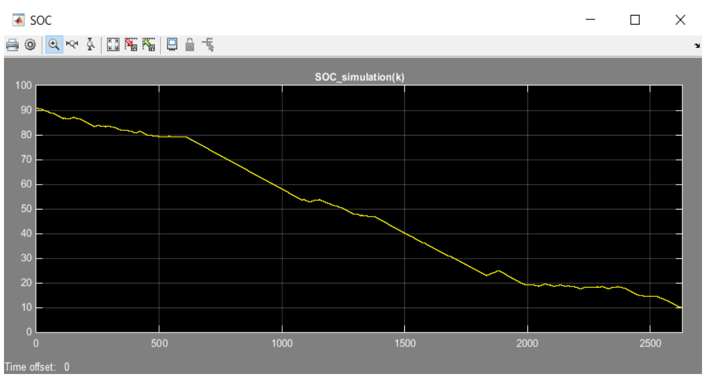


Terminal Voltage Calculation

- Create the OCV-SOC curve using a 10^{th}-order polynomial:



- Profiles of the SOC and Vt:

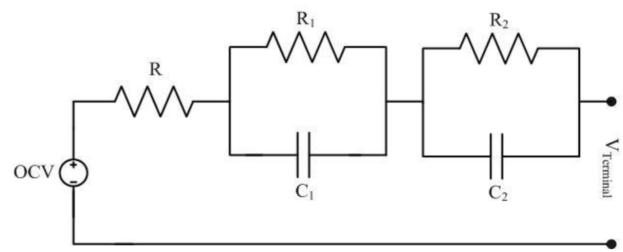


Assignment

- The 2nd-order R-RC-RC model is defined as follows. Parameters of the model are presented in the table and the OCV-SOC relationship is similar to one used by the 1st-order R-RC. Design a 2nd-order R-RC-RC model in Simulink and compare the simulated terminal voltage with the measured one.

$$\begin{bmatrix} V_{1,k+1} \\ V_{2,k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 & 0 \\ 0 & 1-\dfrac{\Delta t}{R_2 C_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} \\ \dfrac{\Delta t}{C_2} \\ -\dfrac{\eta \Delta t}{C} \end{bmatrix} i_k,$$

$$V_{\text{Terminal},k} = OCV(z_k) - V_{1,k} - V_{2,k} - R i_k,$$



Numeric values of parameters for the R-RC-RC model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	28730.04 (Amp.s)
modeling resistance, R_1	0.00349 (Ohms)
modeling capacity, C_2	7583.62 (Amp.s)
modeling resistance, R_2	0.00664 (Ohms)
internal resistance, R_0^+	0.03731 (Ohms)
internal resistance, R_0^-	0.02564 (Ohms)
sampling time, Δt	0.062 (s)

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Any Question?

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