A Tutorial on Modeling Li-Ion Cells Using Equivalent Circuit Models

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Abstract

This tutorial is prepared to teach basic concepts for modeling Li-Ion batteries. It helps students create and run a 1st-order R-RC model with and without hysteresis in MATLAB-Simulink. It starts with a general background about main definitions and terminologies, operating principles of Li-Ion cells, and different modeling approaches. It moreover describes a typical battery test setup and a number of test procedures that are usually used for modeling dynamics of Li-Ion cells. The tutorial continues with descriptions of the 1st-order R-RC model with and without hysteresis, creating the model in MATLAB-Simulink, running test procedures, and parameterization of the model using the genetic algorithm. It is assumed that the reader is familiar with the basic concepts of control systems, continuous and discrete-time simulation, genetic algorithm and MATLAB-Simulink environment. The Simulink model of the 1st-order R-RC model with and without hysteresis is described in details.

1. Introduction

Lithium-Ion batteries are increasingly used in energy storage devices for applications such as electric vehicles, cell phones, medical devices, laptops, etc. This is due to their high energy density, durability, safety, lack of hysteresis, and slow loss of charge when not in use. In order to improve the performance of Li-Ion batteries and increase their safety and efficiency, accurate management, monitoring, and control are required. In this regard, battery management systems are designed in order to estimate quantities representing battery's operating conditions (e.g. state of charge (SOC), state of health (SOH), etc.) and at the same time to prevent the battery from working under dangerous situations. Accurate modeling of electro-chemical reactions occur inside a cell is the primary step for building a battery management system.

A battery consists of a number of connected cells that converts the stored chemical energy into the electric energy. Each cell consists of two electrodes and an electrolyte solution they operate such that oxidation-reduction reactions generate the electric energy. There are two types of batteries including: 1) the primary battery that is not rechargeable and involves irreversible chemical reactions (e.g. energizer cells); 2) the secondary battery that is chargeable and involves reversible chemical reactions (e.g. Li-Ion cells, Lead-Acid cells, etc.).

In continuation, some terminologies and definitions for a battery cell are reviewed:

Battery (Cell, Modules, and Packs): Electric vehicles use a high voltage battery pack that contains two or more modules, and each module consists of a number of cells. A cell is the smallest unit connected in parallel or in series into a module. A module is also connected to other modules in parallel or in series to construct one pack. Voltage, current, and temperature are measured by sensors from each module. Figure 1 presents a battery cell, module, and pack used by Nissan Leaf.

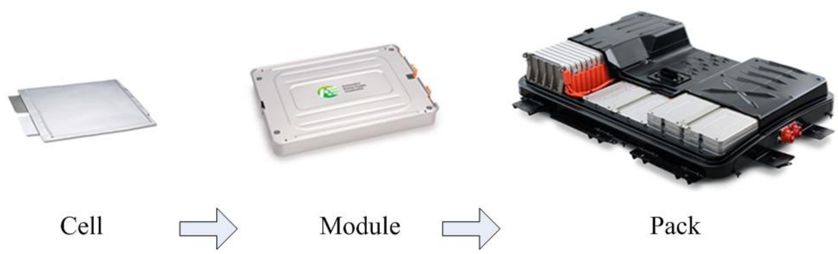


Figure 1: The battery cell, module, and pack used by Nissan Leaf [1]

Charge Rate (C-Rate): It represents the rate at which the battery is charged or discharged with respect to its maximum capacity. In this context, 1 C-rate means that the discharge current will discharge the battery in 1 hour [1].

Open-Circuit Voltage (V_{ocv}): It is the voltage across the two ends of a cell when no load is applied.

Terminal Voltage ($V_{Terminal}$): It is the voltage across the two ends of a cell when a load is applied.

Nominal Capacity (C): It is the total Amp-hour that can be drawn from a healthy battery when the battery is discharged at a certain C-rate. The nominal capacity is calculated by multiplying the current (in Amps) by the discharge time (in hours) [1].

Internal Resistance (R): It shows the resistance within the battery components. As R increases, since it converts the charging energy into heat, the battery efficiency decreases [1].

Cycle Life: It is the number of full charging/discharging cycles before the cell's capacity drops to 80% of its initial capacity. The cycle life is a function of the operating temperature and the C-rate.

State of Charge (SOC): It represents the battery's current capacity as a percentage of its maximum capacity. It may be calculated using coulomb counting (or current integration) [1].

State of Health (SOH): It is an indicator that reflects the general condition of a battery (a cell or a pack), compared to its ideal condition [2].

2. Operating Principles of Li-Ion Batteries

A battery converts the chemical energy into the electrical energy and vice versa. The terminal voltage of a battery is generated by the energy of chemical reactions occur inside each cell. Figure 2 shows the basic setup a battery during charging and discharging. A battery in general consists of the anode, the cathode, the electrolyte, the separator, and current collectors. Anode is the source of reduction reactions during charge and oxidation reactions during discharge. Anode is usually made of a mixture of Carbon (e.g. Li_xC_6). Cathode is the source of oxidation reactions during charge and reduction reactions during discharge. It usually made of metal oxides (e.g. $LiMn_2O_4$). The separator is a porous membrane separating the two electrodes physically. It facilitates ion flow from one electrode to the other and prevents electrons from crossing. Electrolyte is an electrically conducting solution and generates ions that are necessary to support electro-chemical reactions. Electrolyte is usually made of liquid, polymer, or solid materials (e.g. $LiPF_6$) [3].

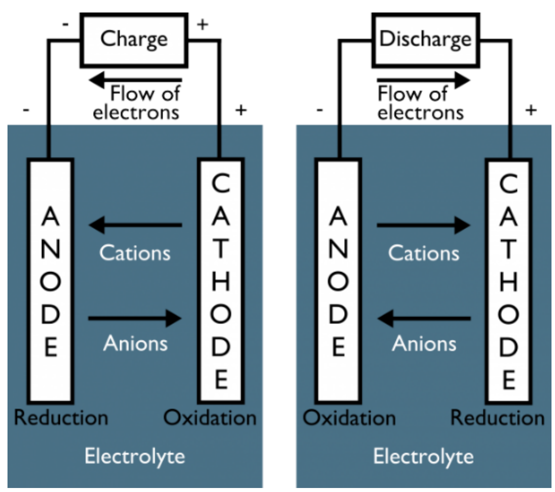


Figure 2: The basic setup of a battery during charging and discharging [3]

In the Li-Ion cells, charging and discharging processes occur by transferring lithium ions between the anode and the cathode across the solution, and electrons through the current collectors. During discharge, the negative electrode gives up electrons to the external circuit. Hence, the negative electrode is oxidized, where oxidation involves loss of electrons, or increase in the oxidation state. In contrast, during charge, the negative electrode receives electrons from the external circuit. This in turn results in a reduction of the species by gaining electrons, or equivalently a decrease in its oxidation state. In this context, reduction is referred to as the reaction in which atoms gain electrons and oxidation is referred to as the reaction in which atoms give up electrons (see Figure 2) [3].

3. Different Modeling Approaches

There exists a large number of methods for modeling batteries. These methods can generally be classified into three main groups as: 1- empirical models, 2- equivalent circuit models, and 3- electro-chemical models. A choice among these models is actually a trade-off between the modeling complexity, accuracy, and its computational cost. Modeling is performed by conducting a series of tests to determine the battery's nominal capacity, the relationship between the open-circuit voltage and the SOC, and cycling tests. In cycling tests, the cell dynamics are excited by various input cycles with variable amplitudes and frequencies. After selecting the model, its unknown parameters need to be identified using an optimization method against actual and measured input-output data. The optimization method tunes model parameters by minimizing the output error that is the difference between the model output and in this case terminal voltage measurement at each time step.

Empirical models or black-box models simulate the terminal voltage behavior of Li-Ion batteries without the need for considering the underlying physics or any electro-chemical reactions that may happen within the cells. These models are mainly based on a series of math functions with unknown parameters. Values of these parameters can be calculated using a set of input-output data and an optimization method. The optimization method calculates the unknown parameters by minimizing the output error that is referred to as the difference between the simulated and the measured terminal voltage. Equivalent circuit models use lumped-element components such as resistors and capacitors to simulate dynamics of a cell. Based on the different levels of modeling complexity, they may include first-order, second-order, or third-order resistor-capacitor elements in addition to an element that shows the hysteresis effect. Values of these elements (e.g. resistances and capacitances) are calculated using input-output data as well as numerical optimization.

In contrast, the electro-chemical modeling techniques consider the electro-chemical reactions happening inside the battery cells. They simulate the internal electro-chemical dynamics of the cell using a set of partial differential equations. Electro-chemical modeling is the most accurate approach among these three approaches, while it is computationally more expensive. Several techniques have been reported in the literature in order to simplify electro-chemical models and apply them for real-time implementations. However, the choice among these three modeling approaches is a compromise between model complexity, accuracy, and computational cost. Figure 3 compares pros and cons of each approach for modeling dynamics of a cell.

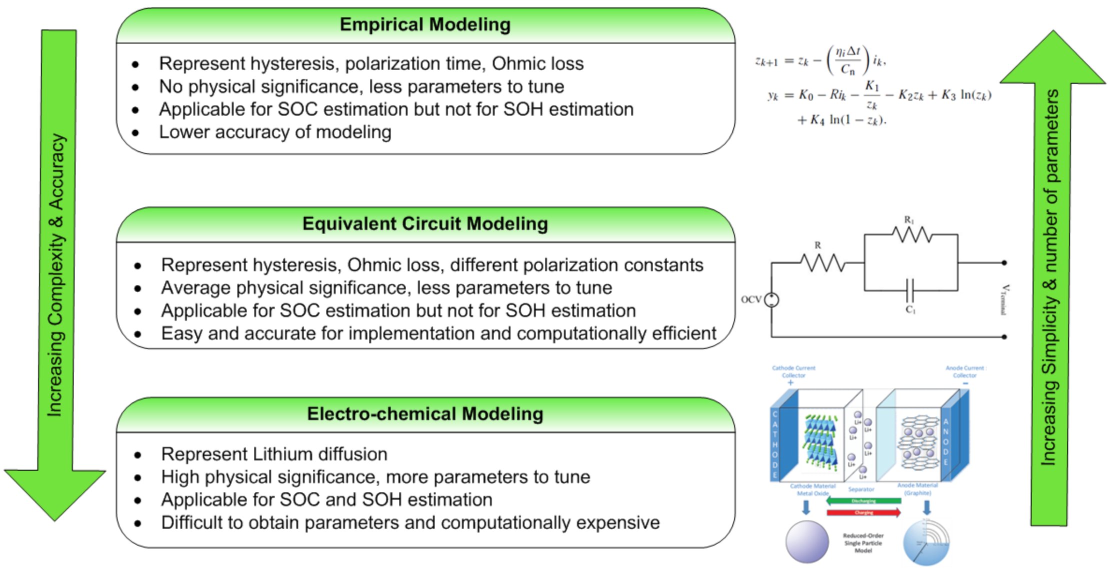


Figure 3: Pros and cons of the three approaches for modeling a battery cell

4. A Typical Battery Test Setup and Test Procedures

In order to conduct experiments on a Li-Ion cell, a battery test setup is designed and built at the Centre for Mechatronics and Hybrid Technology (CMHT). Data collected from these tests are used for modeling and parametrization of the battery cell, and later on, for the SOC estimation in real-time. The guideline for these tests are prepared based on the “Battery test manual for plug-in HEVs” report that was published by the U.S. Idaho National Laboratory in 2008 [4]. Figure 4 presents main elements of a typical battery test setup. The battery test setup includes a Li-Ion cell, an environmental chamber, a power supply, a data acquisition system, a current sensor, thermocouples and a safety circuit. The battery holder is used to enclose the battery and three thermocouples. The power supply is able to provide ±6V and ±150 A. It is connected directly to the Li-Ion cell for charging and discharging and controlled using the computer via connections to the data acquisition chassis. Voltage, current, and temperature measurements are acquired at a sampling frequency of 16 Hz.

Figure 4: Main components of a typical battery test setup

The power supply connections is shown in Figure 5. The 8-slot Ethernet chassis used is the NI cDAQ-9188. Analog Input modules, Analog Output modules and thermocouple modules are incorporated within the chassis. A safety circuit (see Figure 4) was used to cut off power from power supply to the battery if MAX/MIN allowable voltage, current and temperature limits were exceeded. The allowable limits were set using potentiometers. The environmental chamber used is able to vary temperature between -66°C to 177°C. The workspace inside the chamber is 8 cubic feet. Figure 6 presents the Li-Ion cell inside the environmental chamber. The environmental chamber uses two 2.5 HP compressors in order to change temperature within its chamber. Current measurements are taken using a highly accurate hall-effect current sensor (LEM IT 200-S Ultra Slab). The thermocouple connections are shown in Figure 7. Four Omega type-T thermocouples were employed in order to monitor and record temperature. Three thermocouples were used for monitoring battery temperature and one was used for ambient temperature measurements.

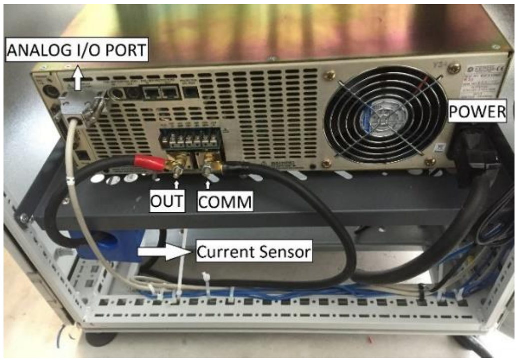


Figure 5: Power supply connections

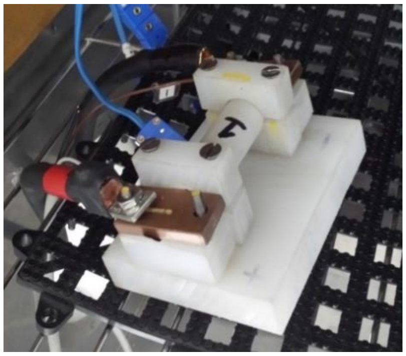


Figure 6: The Li-Ion cell inside the chamber

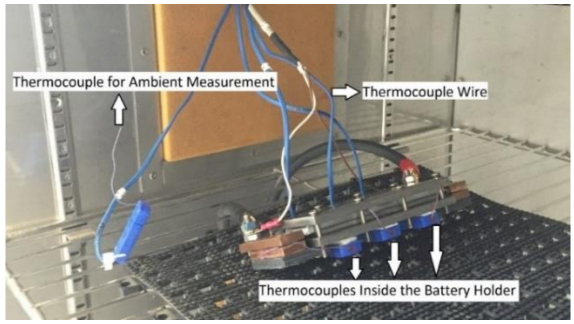


Figure 7: Thermocouple connections inside the chamber

A fresh rechargeable Li-Ion cell is used for the purpose of experiments, modeling, parametrization, and SOC estimation. It is a cylindrical cell with Li-Ion chemistry. Table 1 presents main characteristics of the cell based on the manufacturer's data sheet.

Table 1: Physical characteristics of the Li-Ion cell provided by the manufacturer

Parameter	Condition	Numeric Value
Capacity	Minimum	2.00 Ah
Cell Voltage	Nominal	3.6 V
	Charge	4.2 V
	Discharge	2.0 V
Charge Current	Standard	2.0 A
	Maximum	6.0 A
Charge Time	Standard	1.5 hr
Discharge Current	Maximum	20 A

A number of tests is conducted to specify cell's baseline characteristics such as capacity, internal resistance, open-circuit voltage versus the SOC, as well as aging effects. The first three tests are referred to as the reference performance tests and are performed on a fresh cell at a controlled room temperature of 25°C. The fourth test is the aging test and is conducted to study the effect of battery degradation due to multiple charging/discharging. The guideline for these tests are prepared based on a report published by the U.S. Idaho National Laboratory [4]. These tests are described and conducted as follows:

1- **The Static Capacity Test:** The static capacity test is conducted to measure the cell nominal capacity in Ampere hours at a constant current discharge rate. It provides a baseline for measuring the nominal capacity of a fresh cell. This test applies based on the constant-current constant-voltage mode, as follows *[4]*:

a) Charge the Li-Ion cell at 1 C-rate (2 A) in a constant-current constant-voltage mode to reach the fully charged state. The cell will be fully charged at 4.2 V and the test will stop when the current is at 0.02 C (0.04 A);

b) Disconnect the cell and leave it to rest with no load for one hour in order to stabilize the voltage and the current;

c) Discharge the cell at 1 C-rate with a constant current until the cell voltage hits the cell minimum voltage limit at 2 V;

d) Disconnect the cell and leave it to rest with no load for one hour.

In this case study, the nominal capacity of the battery cell is obtained by conducting the static capacity test and is equal to 7380 *A.sec*.

2- The OCV-SOC Characterization Test: The OCV-SOC characterization test is conducted to characterize the open-circuit voltage (OCV) as a function of the state of charge (SOC) variations. In order to minimize the cell dynamics as well as its Ohmic loss effects that is due to the internal resistance, very small C-rates (C/20, C/15) are applied for the OCV-SOC characterization test. In this context, by performing this experiment with very small C-rates, the measured terminal voltage is assumed to be equal to the open circuit voltage. This test is similar to the capacity test but is conducted at a very low C-rate. The OCV-SOC characterization test is performed, as follows [4]:

a) Charge the cell in a CCCV mode until reaching the maximum voltage limit at 4.2 V.

b) Discharge the cell at the constant current mode with 1 C-rate until the voltage reaches the minimum voltage limit at 2 V.

c) Disconnect the cell from the chamber and leave it and other components of the setup to rest with no load. This causes that the cell remains at a zero state of charge.

d) Charge the cell at a very small C-rate of C/15 (0.1333 A) in a constant-current constant-voltage mode until it hits the maximum voltage of 4.2 V. The cell is left to rest for one hour.

e) Discharge the cell at the same rate of C/15 (0.1333 A) until and the battery hits the minimum voltage of 2 V.

The importance of the OCV-SOC characterization test is for creating the measurement model and accordingly estimating the terminal voltage. The SOC-OCV characterization test was separately conducted for charging and discharging with a low C-rate equal to C/15 (0.1333 A). Using a law C-rate for charging and discharging causes that the voltage drop due to the internal resistance of the cell will be negligible. Moreover, this minimizes battery dynamics, and hence, the measured terminal voltage can be considered as the open circuit voltage. To create the measurement model, the OCV profile needs to be formulated as a function of the SOC. Thereafter, the charging and discharging curves were averaged to obtain a single fixed profile that correlated the OCV values to SOC variations. A polynomial function is then used for model parameters fitting of the averaged profile. In this case study, the SOC-OCV characterization test is separately conducted for charging and discharging with C/15-rate. The average of the $OCV(z_k)$ curve for charging and discharging is calculated. Thereafter, the MATLAB curve fitting command is used to fit the average profile with a 10th-order polynomial function. Table 2 shows numeric values of the corresponding eleven coefficients. Figure 8 presents three profiles including the $OCV(z_k)$ curve for charging, the $OCV(z_k)$ curve for discharging, and their average curve.

Table 2: Numeric values of the 10th-order SOC-OCV polynomial represented by:

$$OCV(z_k) = p_{10}z_k^{10} + p_9z_k^{9} + p_8z_k^{8} + p_7z_k^{7} + p_6z_k^{6} + p_5z_k^{5} + p_4z_k^{4} + p_3z_k^{3} + p_2z_k^{2} + p_1z_k + p_0$$

Coefficient	p_1	p_2	p_3	p_4	p_5	p_6
Numeric value	-10150.680	54373.424	-125525.419	163388.70	-131706.215	67987.955
Coefficient	p_7	p_8	p_9	p_{10}	p_{11}	
Numeric value	-22460.647	4613.866	-554.993	35.657	2.529	

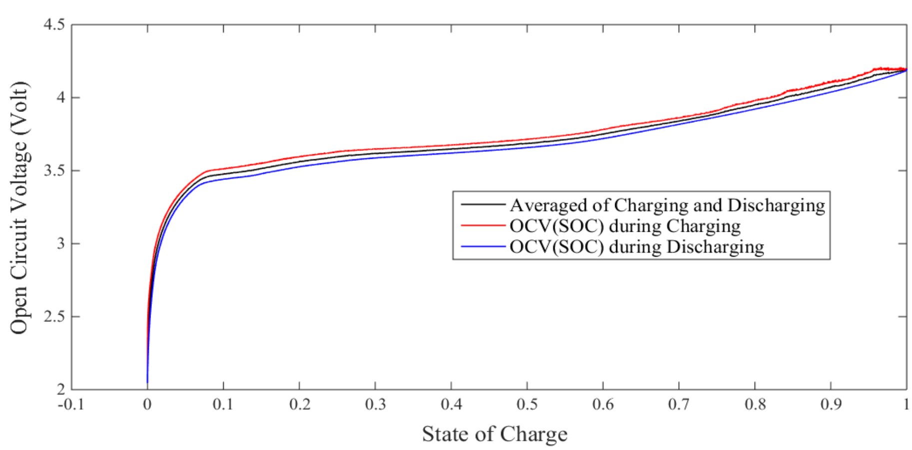


Figure 8: Profiles of the OCV-SOC for charging, discharging, and their average

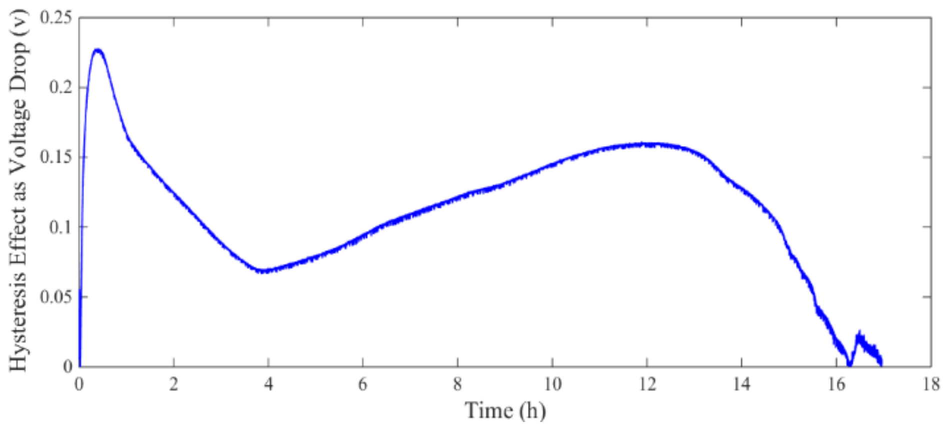


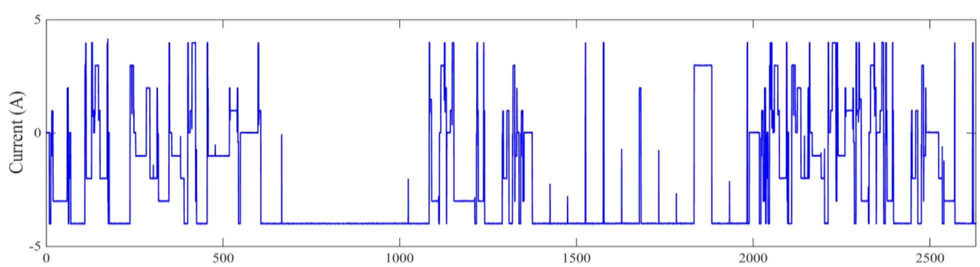
Figure 9: Profile of the voltage drop due to hysteresis using the OCV-SOC test

It is deduced from Figure 8 that the cell behavior is not identical for charging and discharging. It is because of the hysteresis effect that results in loss of energy. The amount of voltage drop by the hysteresis effect may simply be obtained by subtracting the SOC-OCV curve for discharging from the one obtained for charging. Figure 9 shows the voltage drop due to the hysteresis effect during the OCV-SOC test. This profile may be used to model the hysteresis effect.

3- The Driving Cycle Test: In this research, two current cycles are used in order to excite the cell's dynamics. They include: 1- a current cycle provided by the manufacturer, and 2- a current cycle obtained by the urban dynamometer driving schedule (UDDS) *[5, 6]*. The terminal voltage is measured as an output. Figure 10 presents profiles of the applied input current cycle i_k, the measured terminal voltage $V_{t,k}$, and the SOC. The SOC profile is calculated though numeric integration of i_k over time (coulomb-counting), using the following equation *[7]*:

$$z_{k+1} = z_k - \frac{\eta \Delta t}{C_n} i_k , \tag{1}$$

where z_k is the state of charge, η is the Columbic efficiency, C_n is the cell's nominal capacity, and Δt is the sampling time.



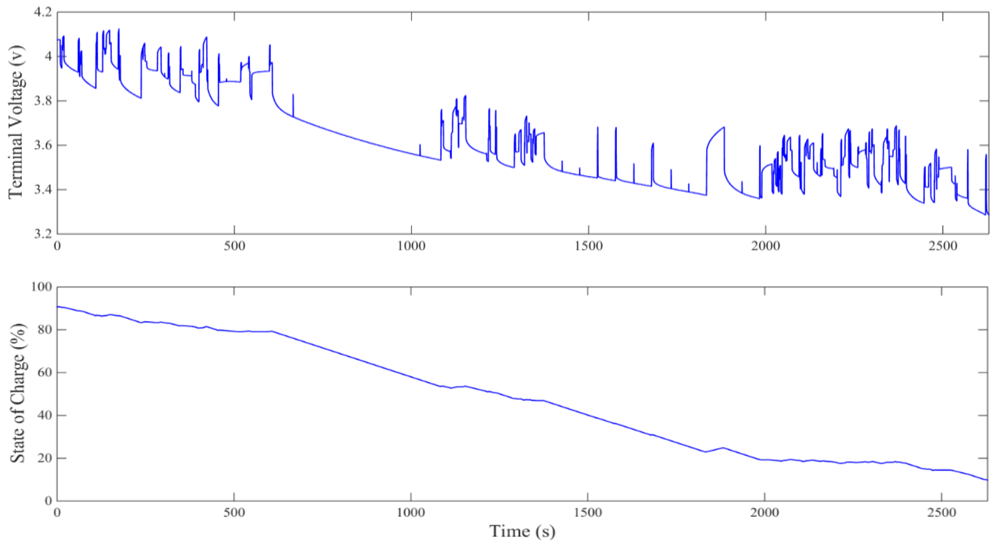


Figure 10: Profiles of the input current, terminal voltage, and the SOC for the Li-Ion cell

5. Optimization Using the Genetic Algorithm

Newton-Raphson and its variants are mainly based on using local information for optimization. They use the function value and its derivative with respect to the optimized parameters to find the local maximum or minimum. The Newton-Raphson approach fails, since a local method can only find local extrema [8]. Figure 11 presents the main concept of finding local minima by means of the Newton-Raphson approach. In order to come up with this issue, the evolutionary algorithms have been developed and implemented in literature. In this report, the genetic algorithm is briefly described as the most famous evolutionary algorithm for optimization. This algorithm is later used for parametrization of the 1st-order R-RC model.

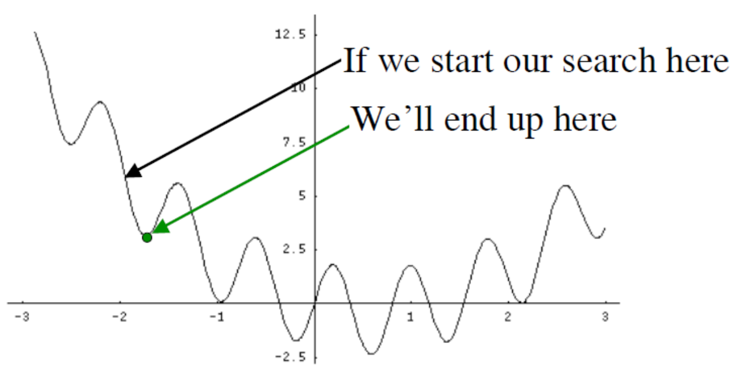


Figure 11: Main concept of finding local minima by the Newton-Raphson [8]

Genetic algorithms are referred to as a class of stochastic search strategies that are modeled using evolutionary mechanisms. A genetic algorithm is capable of optimizing nonlinear systems with a large number of variables. It is an iterative process that starts with a population of randomly generated individuals. The population of each iteration is referred to as the generation and the value of the objective function for the optimization problem is referred to as the fitness. The fitness value for individuals of the population is calculated by each generation. The optimization process applies a random search technique to select the more fit individuals from the current population. Thereafter, the individual populations are modified and possibly randomly modified to create a new generation. This modification is referred to as the recombination. The genetic algorithm ends when either the maximum number of iterations meets, or an appropriate value for the fitness function is reached. A genetic algorithm may be summarized into five main steps as: 1- Select parameters for optimization, 2- Create initial population of individuals, 3- Calculate fitness value of each individual, 4- Apply selection criteria and random search to select survivors, 5- Create a set of new individuals randomly [9]. Figure 12 presents one cycle of the iterative genetic algorithm.

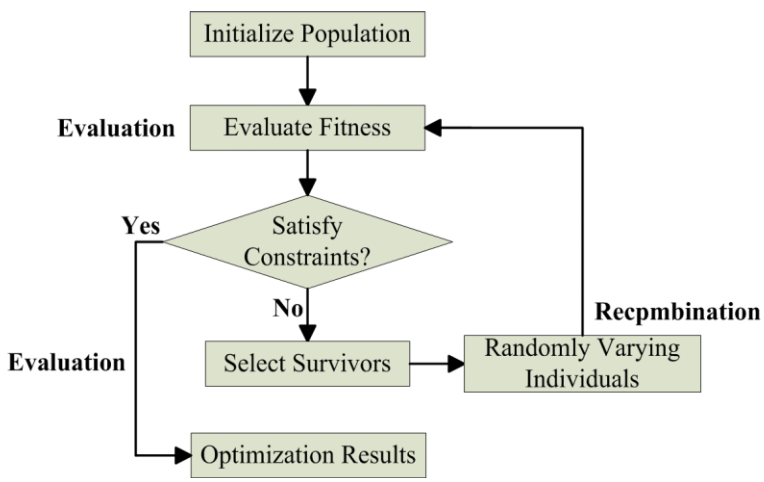


Figure 12: one cycle of the iterative genetic algorithm (upgraded from [8])

In order to model the Li-Ion cell, the model structure is obtained from the equivalent circuit (e.g. 1^{st}-order R-RC) approach, where its parameters are unknown. The three aforementioned test procedures are run in order to find physical characteristics of the cell. Input-output (current-terminal voltage) data are generated from the excitation tests. The genetic algorithm applies using the 1^{st}-order R-RC model and input-output data to identify parameters of the model. It minimizes the error that is the difference between the simulated and the measured output. For this case study, the population size and the generation number are respectively set to 200, and 15. In order to increase the optimization accuracy, these numbers need to be larger.

6. Modeling Li-Ion Cells Using the Equivalent Circuit Approach

The equivalent circuit modeling approach is one of the most popular approaches for modeling Li-Ion batteries. It is because a circuit model may be rather simple, e.g. only has a voltage source and a variable resistance, or may be complex given local conditions in a spatially-resolved model [10]. This approach uses a group of resistors and capacitors, where their magnitudes are obtained using an optimization method. The optimization method employs a random search at each time step to extract the parametric values such that the error between the measured terminal voltage and the simulated one is minimized. The main advantage of the equivalent circuit approach for modeling is its capability for real-time applications with an acceptable performance. However, the main disadvantage of this approach is its limitation for considering the electro-chemical reactions that take place internally inside the cell. This limitation prevents it from modeling some physical behaviours including the power fading, the capacity fading and also the aging effect [10]. In this report, the 1st-order R-RC model with and without the hysteresis effect is described in details. Thereafter, it is implemented for modeling dynamics of a Li-Ion cell using real-world data. Figure 13 presents a general overview of the main steps for modeling a battery cell.

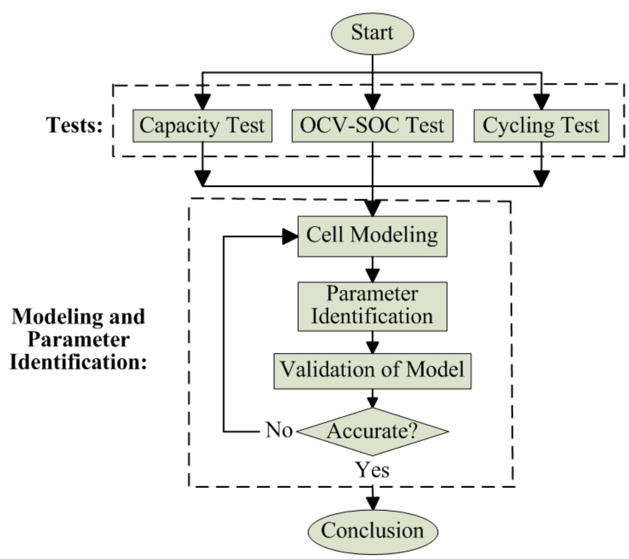


Figure 13: Main steps for modeling a battery cell

6.1. The 1st-Order R-RC Model

The 1st-order R-RC model is the simplest equivalent circuit model in which the battery dynamics are modeled using three elements including: 1) the internal resistance R, 2) the modeling resistance R_1, and 3) the modeling capacitance C_1. Note that the internal resistance R represents the Ohmic resistance of a battery cell. It has two values, R^+ for a positive input current, and R^- for a negative input current. Figure 14 shows a circuit diagram for the 1st-order R-RC model. The state-space representation of the 1st-order R-RC model is given by [11]:

$$\begin{bmatrix} V_{1,k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} \\ -\dfrac{\eta \Delta t}{C} \end{bmatrix} i_k, \tag{2}$$

$$V_{\text{Terminal},k} = OCV(z_k) - V_{1,k} - R i_k, \tag{3}$$

where R_1 and R respectively denote the modeling and the internal resistance. C_1 is the capacitance across R_1, and C is the nominal capacitance of the entire cell. i_k is the input current, z_k is the state of charge, Δt is the sampling period, and η is the cell Columbic efficiency. $V_{\text{Terminal},k}$ is the model output that is the terminal voltage across the two ends of the cell, and $OCV(z_k)$ is the open-circuit voltage relationship as a polynomial function of z_k [11]. This polynomial function is given by:

$$OCV(z_k) = p_{10} z_k{}^{10} + p_9 z_k{}^9 + p_8 z_k{}^8 + p_7 z_k{}^7 + p_6 z_k{}^6 + p_5 z_k{}^5 + p_4 z_k{}^4 + p_3 z_k{}^3 + p_2 z_k{}^2 + p_1 z_k + p_0, \tag{4}$$

where coefficients p_0 to p_{10} are represented in Table 3.

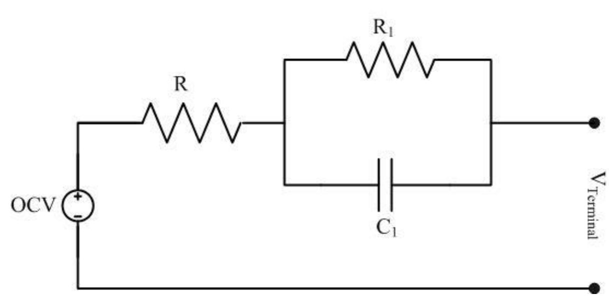


Figure 14: The 1st-order R-RC model of a battery cell [11]

The 1st-order R-RC model has two state variables that are the state of charge (SOC) z_k and the voltage $V_{1,k}$ that is the voltage across resistance R_1. The 1st-order R-RC model has four unknown parameters that are R_1, C_1, R^+ and R^-. Numeric values of these parameters are calculated using the input-output data and the genetic algorithm. Note that the population size and the generation number are respectively set to 200, and 15. The sampling time is equal to 0.062 sec, and the cell columbic efficiency η is assumed to be one. Table 3 presents numeric values of the cell physical parameters as well as values of the four optimized parameters. Figure 15 presents profiles of the measured and the modeled terminal voltage.

Table 3: Numeric values of parameters for the R-RC model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	6618.4 (Amp.s)
modeling resistance, R_1	8.62e-05 (Ohms)
internal resistance, R^+	0.0445 (Ohms)
internal resistance, R^-	0.0217 (Ohms)
sampling time, Δt	0.062 (s)

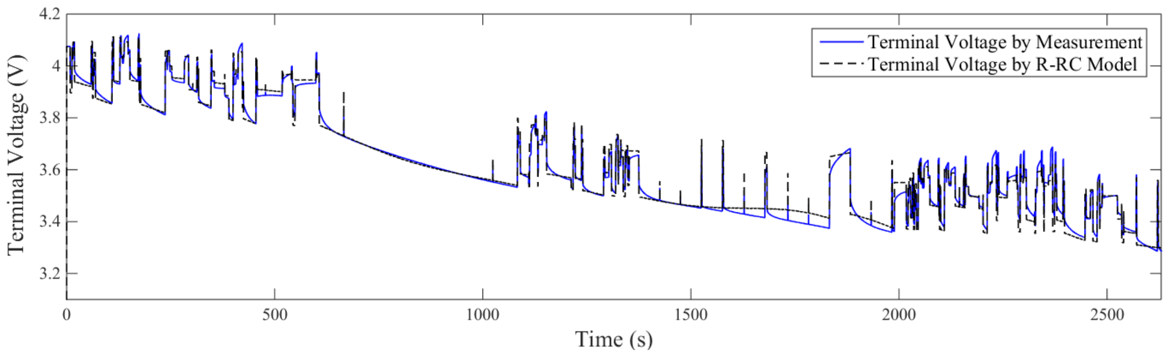


Figure 15: Profiles of the measured terminal voltage and the one obtained by the R-RC model

6.2. The 1st-order R-RC-H Model

The 1st-order R-RC-H model considers an additional element to represent the hysteresis effect and hence, there is an additional state variable referred to as the hysteresis state. This model simulates the Li-Ion battery cell using four elements including: 1- the internal resistance R, 2- the modeling resistance R_1, 3- the modeling capacitance C_1, 4- The hysteresis effect formulated using the maximum polarization constant M, and the hysteresis rate γ. Note that the maximum polarization constant M has two values, M^+ for a positive input current, and M^- for a negative input current. Similarly, the internal resistance R has two values, R^+ for a positive input current, and R^- for a negative input current. Figure 16 presents a circuit diagram for the 1st-order R-RC-H model. The 1st-order R-RC-H model may be represented in the state-space form as follows [6]:

$$\begin{bmatrix} V_{1,k+1} \\ h_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{R_1 C_1} & 0 & 0 \\ 0 & F(i_k) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ h_k \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} & 0 \\ 0 & 1-F(i_k) \\ -\dfrac{\eta \Delta t}{C} & 0 \end{bmatrix} \begin{bmatrix} i_k \\ M(z,\dot{z}) \end{bmatrix}, \tag{5}$$

$$V_{t,k} = OCV(z_k) - V_{1,k} + h_k - R_0 i_k, \tag{6}$$

where h_k denote the hysteresis state, and M is the maximum polarization constant with M^+ and M^- values. $F(i_k)$ represents the hysteresis function and is calculate by [6]:

$$F(i_k) = \exp\left(-\left|\frac{\eta\gamma\Delta t i_k)}{C}\right|\right), \tag{7}$$

where γ is the hysteresis rate, and C is the nominal capacity of the cell.

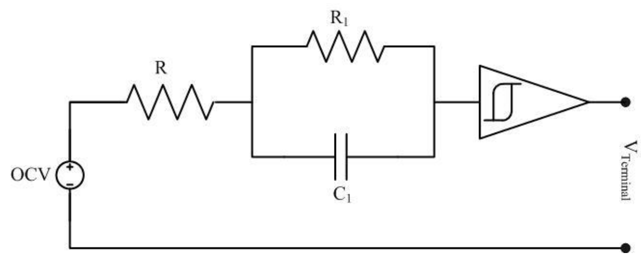


Figure 16: The 1st-order R-RC-H model of a battery cell

In this model, there are three state variables that are: 1- the voltage $V_{1,k}$ across the modeling capacitance C_1, 2- the hysteresis h_k, and 3- the state of charge z_k. Furthermore, there are two inputs including the input current, and the max polarization value (M^+ or M^-) that is selected at each time step based on the direction of the applied current. The output of the system is the terminal voltage $V_{t,k}$ and is calculated by equation (6). The open-circuit voltage $OCV(z_k)$ is also formulated as a 10th-order polynomial function of $SOC(\mathrm{k})$ as listed in Table 2.

Table 4: Numeric values of parameters for the R-RC-H model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	2775.92 (Amp.s)
modeling resistance, R_1	0.0172 (Ohms)
internal resistance, R_0^+	0.0360 (Ohms)
internal resistance, R_0^-	0.0248 (Ohms)
max polarization constant, M^+	0.0178
max polarization constant, M^-	5.384e-05
hysteresis rate, γ	16.676

The 1st-order R-RC-H model has seven unknown parameters include $R_1, C_1, R_0^+, R_0^-, M^+, M^-$, and γ. Numeric values of these parameters are calculated based on the input-output data and the genetic algorithm. Values of other parameters including the simulation parameters are equal to ones used for the R-RC model. Table 4 presents numeric values of the cell parameters for the 1st-order R-RC-H model. Figure 17 presents profiles of the measured and the modeled terminal voltage using the R-RC-H model.

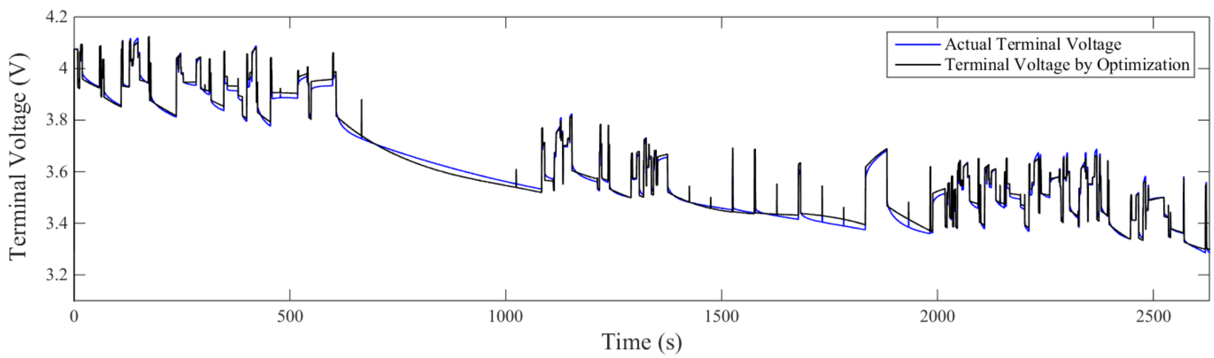


Figure 17: Profiles of the measured and modeled terminal voltage by 1st R-RC-H model

7. General Steps for Creating a Simulink Model

In order to create or run a control system in Simulink, the following steps are required:

1- Start MATLAB

2- Simulink is an extra toolbox that runs on the top of MATLAB. To start Simulink, type "simulink" in the Command Window or click on the Simulink icon. Figure 18 presents how to run Simulink under MATLAB.

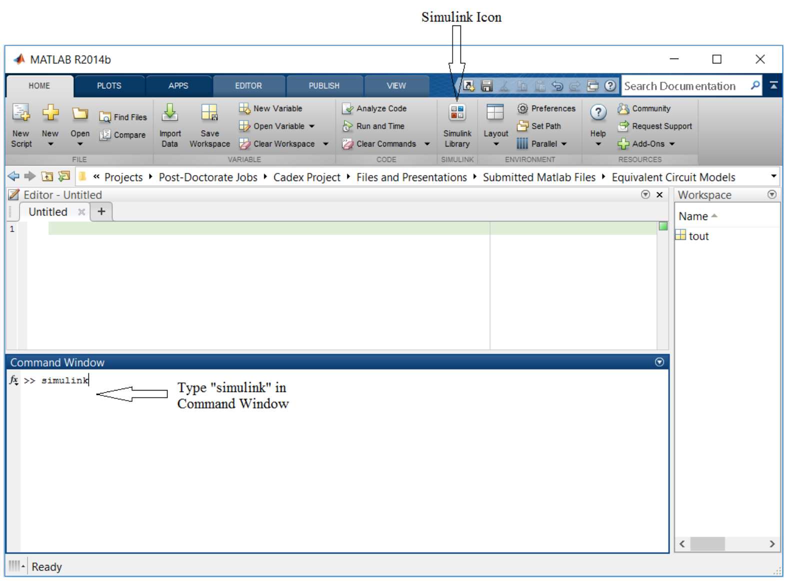


Figure 18: Starting Simulink using the icon or Command Window in MATLAB R2014b

3- The “Simulink Library Browser” now appears, as shown in Figure 19. It provides a library of different blocks, operators, functions, inputs, etc. A typical control system may simply be modeled using the blocks of these library. The Simulink library browser consists of main Simulink blocks and additional blocks. The main block includes the “Commonly Used Blocks”, “Continuous”, “Discrete”, “Math Operations”, “Sources”, “Sinks”, “Lookup Tables”, etc. Additional blocks are designed for more specific applications such as: “Communication Systems Toolbox”, “Computer Vision Toolbox”, “Neural Network Toolbox”, “Simscape”, etc.

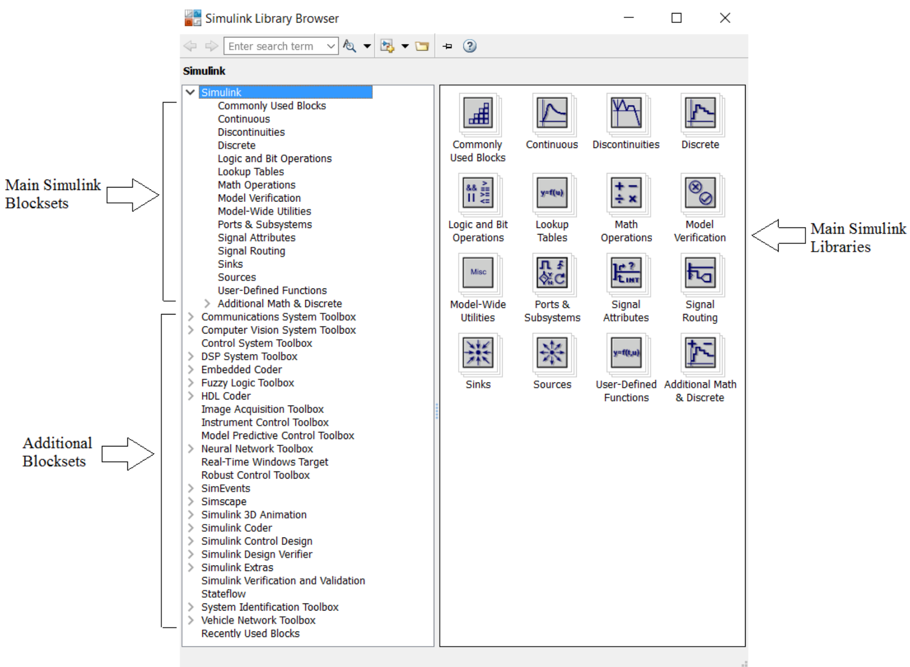


Figure 19: Main features of the Simulink library browser

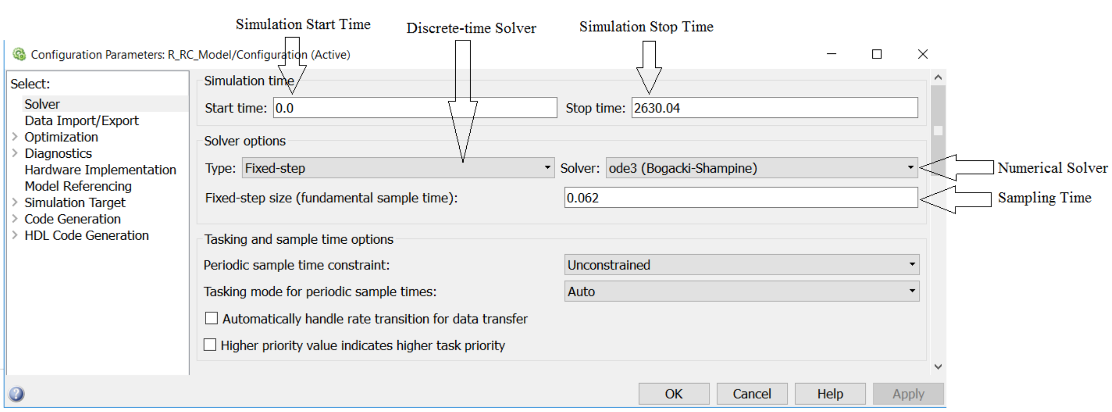


Figure 20: Window of model configuration parameters

4- Open a new Simulink model through: File > New > Model. A new window, named as 'untitled' will appear. It is possible to design a control system by click and drag the required blocks from the "Simulink Library Browser". The blocks can be linked to each other by drawing signals from the output port of each block into the input port of others.

5- Adjust the numeric solver based on the design requirements. To tune the solver, select the following path on the Simulink window: Simulation > Model configuration Parameters. A new window now opens, as presented in Figure 20.

   Note that there are some options for tuning the numeric solver. These options determine the simulation type (e.g. discrete vs. continuous-time simulation), the solver algorithm (e.g. ode4 – Runge Kutta), running time, sampling time, etc. After tuning the solver, the Simulink model can be run by click on the simulation bottom on the top of the main window, as presented in Figure 21.

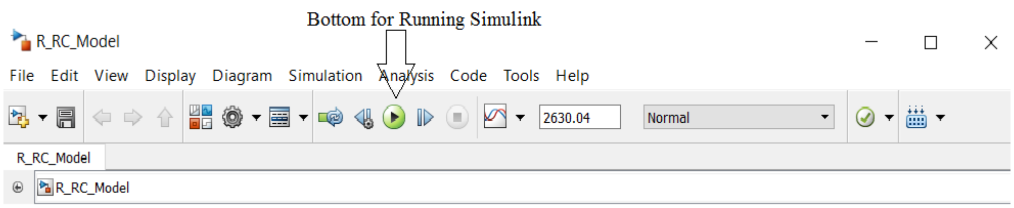


Figure 21: The bottom for running the Simulink model

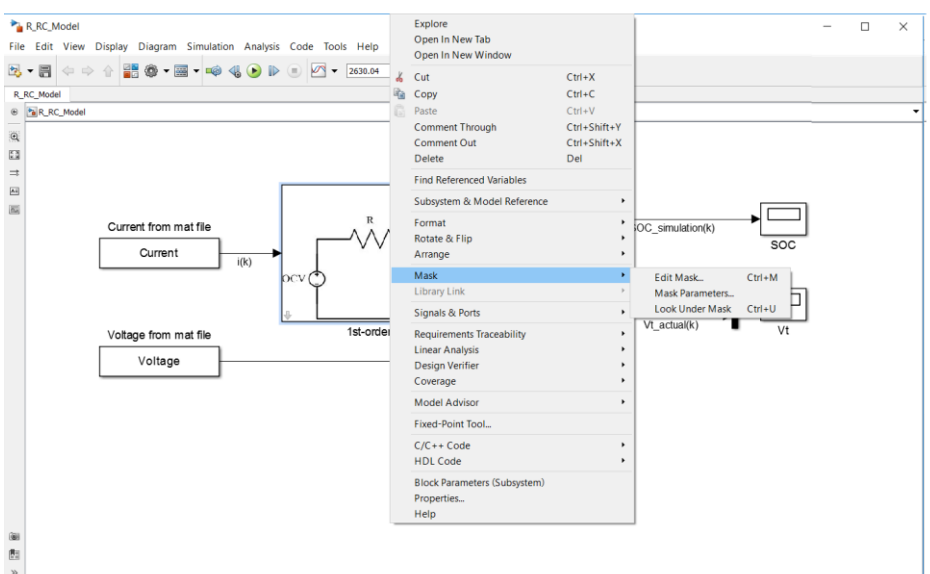


Figure 22: The way to create or edit the mask interface for a Simulink block

Note that it is possible to feed the input parameters to the Simulink model using block masks. A mask is a user interface that is designed to cover a Simulink block such that it has its own parameter dialog box with the block description, parameters, and help texts. To create or edit a mask interface for a Simulink block, right click on the block and select "Mask" in the menu tool bar. Figure 22 shows how to create mask for a block. After creating the mask, by double click on the block, the mask interface appears.

8. Running the 1st-Order R-RC Model in Simulink

Two Simulink models are provided within the supporting files, and are named by R_RC_Model.slx and R_RC_H_Model.slx. They respectively represent dynamics of a Li-Ion cell using a 1st-order R-RC model without and with the hysteresis element. Other supporting files include two JPEG files, and two MATLAB files named by Current.mat, and Voltage.mat. These MATLAB files include the applied input current and the measured terminal voltage, respectively. In order to run Simulink models, all these files should be kept into a folder. The JPEG files are used as cover photos on the Simulink files. Simulink files can only be run using MATLAB R2014b and probably newer versions. Parametric values for these models are fed into models using a mask interface built in Simulink models. The Simulink solver is set to "discrete" that is a fixed-step solver. The sample time for test and simulations is equal to 0.062 sec and the total running period is 2630.04 sec. To run the provided codes by MATLAB R2014b, the MATLAB's path directory needs to be set as the place where the folder contains Simulink models and other supporting files are located.

It is important to note that the input current file "Current.mat" and the output terminal voltage measurement data "Voltage.mat" are captured during the cycling test. These two mat files are fed into the Simulink model though "From Workspace" blocks that are located in "Sources". The Current.mat includes an array with two rows. The first row includes the time sequences and the second row includes the current sequences. Similarly, the Voltage.mat includes an array with two rows. The first row shows the time sequences and the second raw shows the voltage sequences. The input current data is scaled based on the allowable upper and lower limits of the current presented in the manufacturer's data sheet. The nominal capacity of the cell is calculated by conducting the static capacity test. The open circuit voltage – state of charge (OCV–SOC) curve is obtained by conducting the OCV-SOC relationship characterization test.

Parametric values of the 1st-order R-RC model are calculated by running the genetic algorithm optimization method and input-output data. The genetic algorithm identifies parametric values of the 1st-order R-RC model by minimizing the optimization error that is the difference between the measured terminal voltage and the model's terminal voltage. The genetic algorithm method and the provided MATLAB code are later described in this tutorial. Figure 23 presents a picture of the 1st-order R-RC model in Simulink. There are two scopes in the Simulink model and are named as SOC and Vt. The scope block can be found in "Sinks". The two scope blocks are used to plot the SOC and the terminal voltage generated by the model. The Vt-scope compares the model's terminal voltage with the actual one obtained by measurements. A Mux block is used to combine the two voltage signals into a vector that is fed to the Vt-scope. By double clicking on each of Vt or SOC scopes, a diagram appears that shows the Vt or SOC profile.

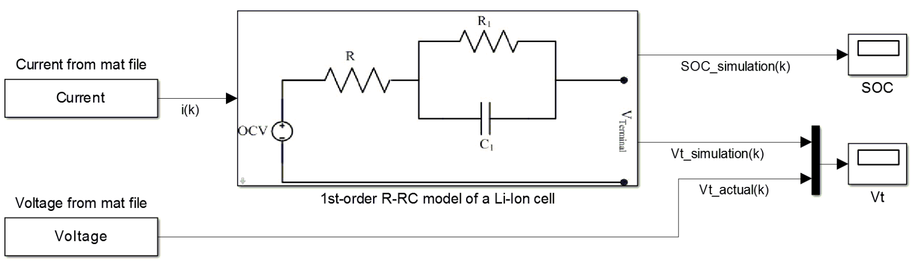


Figure 23: A picture of the 1st-order R-RC model in Simulink

By double clicking on the Simulink model, the designated mask that feeds the parametric values into the model appears. These values of obtained by optimization using the genetic algorithms. Figure 6 shows a picture of the mask and the corresponding parametric values. It is possible to click on the each box and change values of parameters seen on the mask interface.

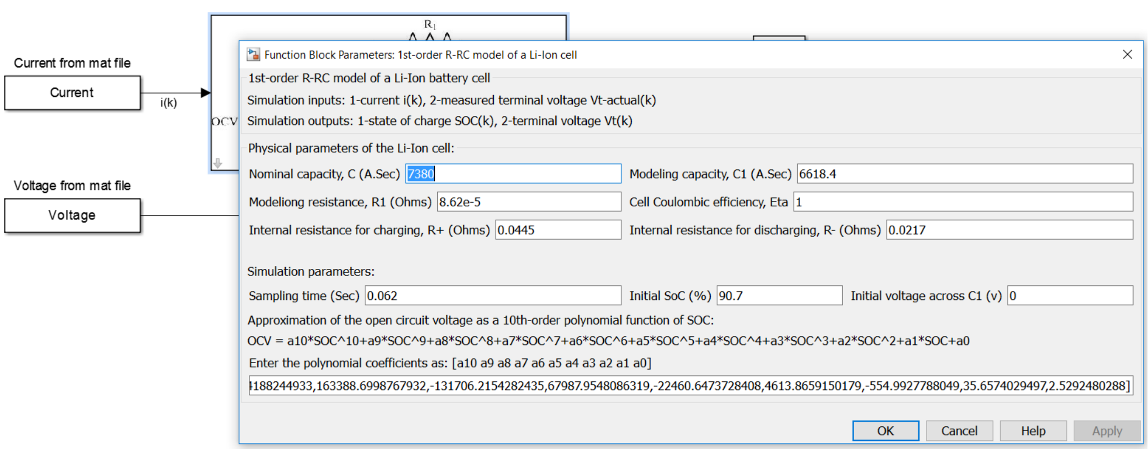


Figure 24: A picture of the 1st-order R-RC model in Simulink

In order to look inside the 1st-order R-RC model and see the constituent elements, click on the arrow sign located in the lower left corner of the Simulink block. Figure 25 shows a picture of constituent elements located inside the 1st-order R-RC block. As presented, the model has one input that is the input current *i(k)*, and two outputs that are the state of charge *SOC(k+1)*, and the terminal voltage $V_{Terminal}(k)$. To unify the direction of the input current with the one used by mathematical equations (2-3), it is multiplied by a gain with the magnitude of -1. The current signal is then applied as an input to the v_1 state equation block and the SOC block. Moreover, the current is multiplied by the internal resistance *R* to construct another element of the output terminal voltage. Note that the value of *R* is different for the charge and discharge, and hence, it depends on the current direction. A switching block is used to select R^+ when the current is positive, and select R^- when the current is negative. Note that the "In1" and the "Out1" block, the "Gain", the "Product", the "Sum", the "Constant", the "Switch", the "Mux", and the "Demux" blocks may all be found in "Commonly Used Blocks".

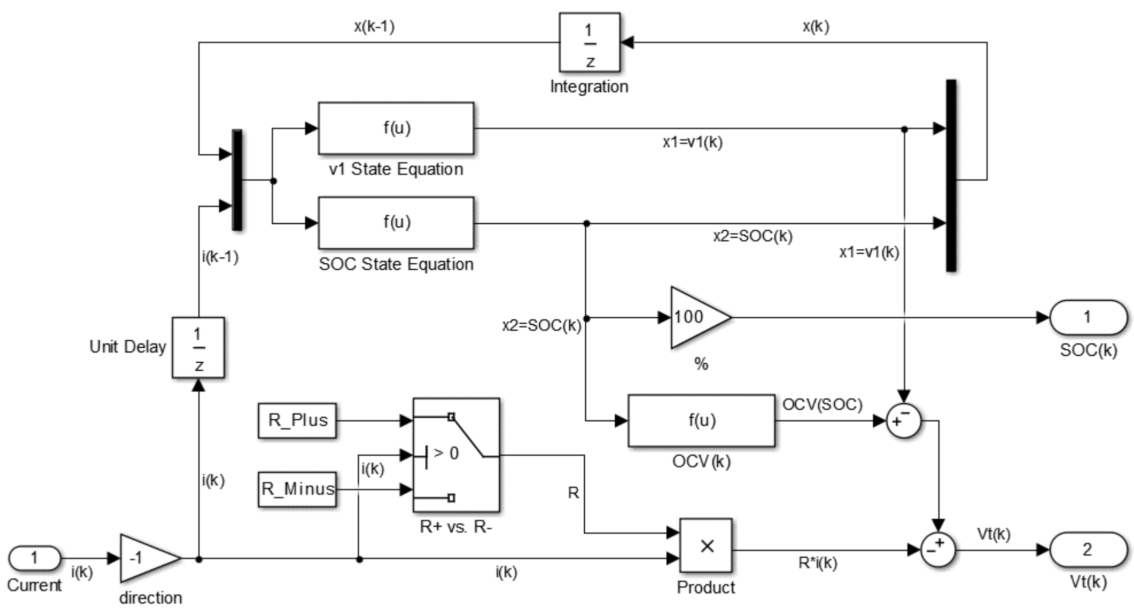


Figure 25: A picture of constituent elements of the 1st-order R-RC model

Two function blocks, named as "Fcn", are used to create the v_1 and the SOC state equations. The Fcn block is located in "User-Defined Functions" and is used to apply mathematical functions on the input signal. Figure 26 presents the formulations that are used by the v_1 and the SOC state equation block. The input *u* to these block has three elements that include state values at the previous step, $v_1(k)$ and *SOC(k)*, and the input current *i(k)*.

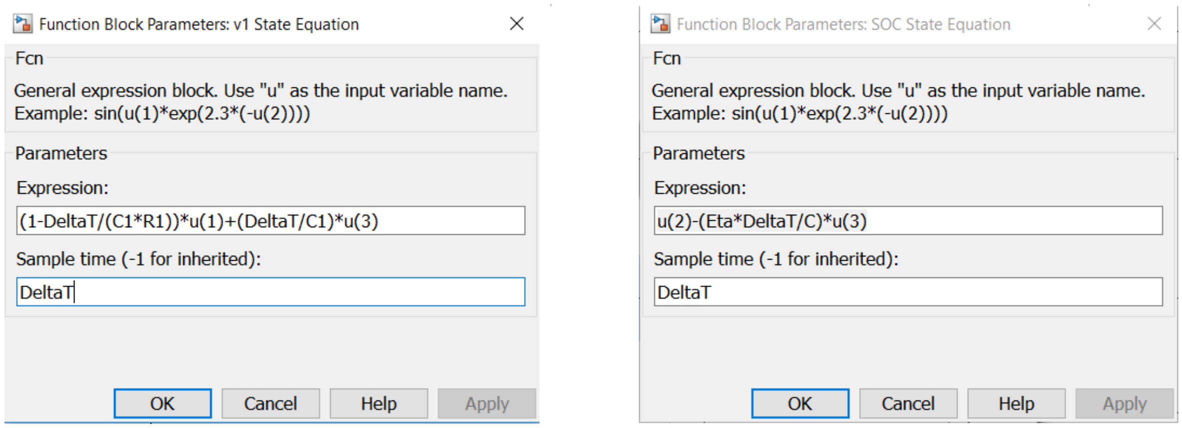


Figure 26: The state equation blocks used by the 1st-order R-RC model

Figure 27 presents a picture of the numeric integrator block in which initial conditions for integration are represented. Figure 28 moreover shows a picture of OCV-SOC function block in which the 10th-order OCV-SOC polynomial relationship is formulated.

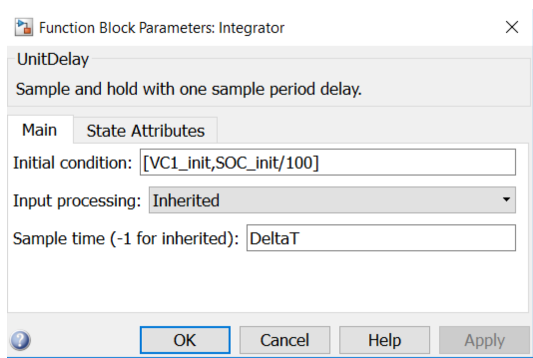


Figure 27: A picture of the numeric integrator block for the 1st-order R-RC model

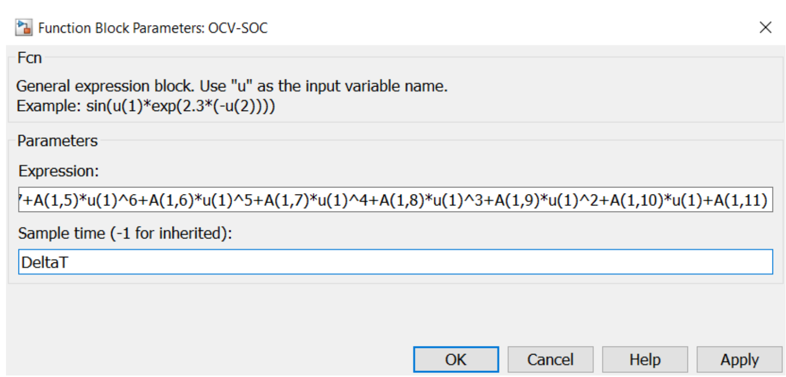


Figure 28: A picture of OCV-SOC function block for the 1st-order R-RC model

Assignment:

A) Given the 1st-order R-RC-H model with parameters of Table 4 in the report, run the model in Simulink and compare the simulated output versus the measured one.

B) The 2nd-order R-RC-RC model is defined as follows:

$$\begin{bmatrix} V_{1,k+1} \\ V_{2,k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1-\frac{\Delta t}{R_1C_1} & 0 & 0 \\ 0 & 1-\frac{\Delta t}{R_2C_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{1,k} \\ z_k \end{bmatrix} + \begin{bmatrix} \frac{\Delta t}{C_1} \\ \frac{\Delta t}{C_2} \\ -\frac{\eta \Delta t}{C} \end{bmatrix} i_k, \tag{8}$$

$$V_{t,k} = OCV(z_k) - V_{1,k} - V_{2,k} - R_0 i_k, \tag{9}$$

The circuit diagram and parametric values of the 2nd-order model are presented by the Figure 12 and Table 5. The OCV-SOC relationship is similar to one used by the 1st-order R-RC. Similar input and output files (Current.mat, and Voltage.mat) can be used for simulation. Design a 2nd-order R-RC-RC model in Simulink and compare the simulated terminal voltage with the measured one. Compare the accuracy of the simulated terminal voltage of the 2nd-order R-RC-RC model with the one obtained by the 1st-order R-RC.

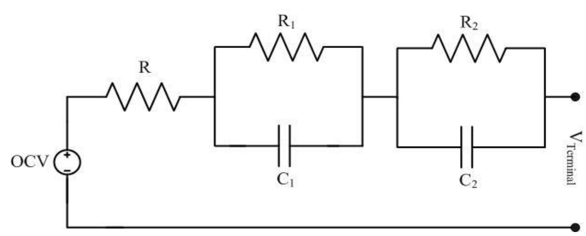


Figure 29: Circuit diagram of the 2nd-order R-RC model

Table 5: Numeric values of parameters for the R-RC-RC model

Parameter	Numeric Value
nominal capacity, C	7380 (Amp.s)
cell Columbic efficiency, η	1
modeling capacity, C_1	28730.04 (Amp.s)
modeling resistance, R_1	0.00349 (Ohms)
modeling capacity, C_2	7583.62 (Amp.s)
modeling resistance, R_2	0.00664 (Ohms)
internal resistance, R_0^+	0.03731 (Ohms)
internal resistance, R_0^-	0.02564 (Ohms)
sampling time, Δt	0.062 (s)

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